Group Connectivity in Products of Graphs

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Abstract

Let G be a 2-edge-connected undirected graph, A be an (additive) abelian group and $A^* = A - \{0\}$. A graph G is A-connected if G has an orientation $D(G)$ such that for every function $b : V(G) \mapsto A$ satisfying $\sum_{v \in V(G)} b(v) = 0$, there is a function $f : E(G) \mapsto A^*$ such that for each vertex $v \in V(G)$, the total amount of f values on the edges directed out from v minus the total amount of f values on the edges directed into v equals $b(v)$. For a 2-edge-connected graph G, define $\Lambda_g(G) = \min\{k :$ for any abelian group A with $|A| \geq k$, G is A-connected$\}$.

Let $G_1 \otimes G_2$ and $G_1 \times G_2$ denote the strong and Cartesian product of two connected nontrivial graphs G_1 and G_2. We prove that $\Lambda_g(G_1 \otimes G_2) \leq 4$, where equality holds if and only if both G_1 and G_2 are trees and $\min\{|V(G_1)|, |V(G_2)|\}$=2; $\Lambda_g(G_1 \times G_2) \leq 5$, where equality holds if and only if both G_1 and G_2 are trees and either $G_1 \cong K_{1,m}$ and $G_2 \cong K_{1,n}$, for $n, m \geq 2$ or $\min\{|V(G_1)|, |V(G_2)|\}$=2. A similar result for the lexicographical product graphs is also obtained.