Power Residue Graphs and Ramsey Numbers

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Whenever we have a prime power, we may use powers in the finite field of that order to define a graphs. For example, quadratic residue graphs appear in several circumstances. This talk has little in the way of new results, but seeks to investigate the role of power residue graphs. It is surprising how many Ramsey numbers have their lower bound determined by one of these graphs.

A long standing question of Jon Folkman, tangentially related to Ramsey numbers, asks for the smallest graph that does not contain a copy of K4, but yet any two coloring of its edges must contain a monochromatic triangle. The original upper bound for such a graph was an astronomical expression. Erdos offered \$100 to lower it below 10^{10}. Various researchers got it down to human size, with the best result, 941, obtained recently by Dudek and Rodl. The structure of the order 941 graph, as well as a candidate graph, suggested by Dudek, that might improve the bound, inspired this talk.