

Generalizing the Lights Out game

Darren Parker

Grand Valley State University

We study the following generalization of the game “Lights Out”. Given a graph G and a set \mathcal{C} with $0 \in \mathcal{C}$, let $\pi : V(G) \rightarrow \mathcal{C}$ be a (not necessarily proper) coloring of $V(G)$. Let $T : \mathcal{C} \rightarrow \mathcal{C}$ be any function. We play the game by toggling the vertices. Each time the vertex v is toggled, we apply T to the colors of the vertex toggled as well as each of its adjacent vertices. The game is won when each vertex has label 0. We prove that if T is a permutation, the game can be reduced to the case $\mathcal{C} = \mathbb{Z}_k$ and $T(x) = x + 1$. We then turn to path graphs, cycle graphs, and complete bipartite graphs and determine when the Lights Out game can be won on these graphs.