An Intuitively Appealing Axiomatization of the median procedure on median graphs

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Abstract

If (X, d) is a finite metric space then we define X^* to be the collection of all finite sequences of elements of X, with repetition allowed. Such a sequence is a *profile* and a *median* of a profile $\pi = x_1, x_2, ..., x_k$ is an element $x \in X$ minimizing $\sum_{i=1}^{k} d(x, x_i)$. Any function $L : X^* \mapsto 2^X - \emptyset$ is called a *consensus function* on (X, d). The *median function*, also called the *median procedure*, is the specific consensus function that maps a profile π to $M(\pi) = \{x \,|\, x \text{ is a median of } \pi\}$. Our concern in this talk will be consensus functions on graphs, specifically 'median graphs', endowed with the geodesic metric.

A *median graph* is a graph for which every profile of length 3 has a unique median. Median graphs have been well studied, possess a beautiful structure and arise in many arenas, including ternary algebra, ordered sets and discrete distributed lattices. Trees and hypercubes are key examples of median graphs.

We show that the median proceedure can be characterized on the class of all median graphs with only three simple and intuitively appealing axioms, settling a question posed implicitly by McMorris, Mulder and Roberts in 1998.

This is joint work with Henry Martyn Mulder.