Every 4-connected line graph of a P_3-dominating graph is hamiltonian-connected

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Abstract

Mathews and Sumner, and Tommasen conjectured that every 4-connected line graph is hamiltonian. We investigate graphs G such that the line graph $L(G)$ is hamiltonian connected when $L(G)$ is 4-connected. Ryjáček and Vrána recently further conjectured that every 4-connected line graph is hamiltonian-connected. In 2001, Kriesell proved that every 4-connected line graph of a claw free graph is hamiltonian connected. Recently, Lai et al showed that every 4-connected line graph of a quasi claw free graph is hamiltonian connected, and that every 4-connected line graph of an almost claw free graph is hamiltonian connected. In 2009, Broersma and Vumer discovered the P_3-dominating (P3D) graphs as a superfamily that properly contains all quasi claw free graphs, and in particular, all claw-free graphs. We in this paper prove that every 4-connected line graph of a P3D graph is hamiltonian connected, which extends several former results in this area.

Keywords: Hamiltonian graphs, claw-free graphs, line graphs, almost claw free graphs, quasi claw free graphs, P_3-dominating graphs