Maximal flat antichains of minimum weight

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We study maximal families $\mathcal{A}$ of subsets of $[n] = \{1, 2, \dots, n\}$ such that $\mathcal{A}$ contains only 2-sets and 3-sets and $A \not\subseteq B$ for all $\{A, B\} \subseteq \mathcal{A}$, i.e. $\mathcal{A}$ is an antichain. For any n, all such families $\mathcal{A}$ of minimum size are determined. This is equivalent to finding all graphs $G = (V, E)$ with $|V| = n$ and with the property that every edge is contained in some triangle and such that $|E| - |T|$ is a maximum, where T denotes the set of triangles in G. The largest possible value of $|E| - |T|$ turns out to be $\lfloor (n + 1)^2/8 \rfloor$. Furthermore, if all 2-sets and 3-sets have weights w_2 and w_3, respectively, the problem of minimizing the total weight $w(\mathcal{A})$ of $\mathcal{A}$ is considered. We show that $\min w(\mathcal{A}) = (2w_2 + w_3)n^2/8 + o(n^2)$ for $3/n \le w_3/w_2 =: \lambda = \lambda(n) < 2$. For $\lambda \ge 2$ our problem is equivalent to the (6,3)-problem of Ruzsa and Szemerédi, and by a result of theirs it follows that $\min w(\mathcal{A}) = w_2 n^2/2 + o(n^2)$.

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