Spanning Connectivity of Line Graphs

Ye Chen*, Zhi-Hong Chen†, Hong-Jian Lai‡*, Ping Li*, Erling Wei§

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Abstract

Spanning connectivity of graphs has been intensively investigated in the study of interconnection networks ([3]). For an integer $s > 0$ and for $u, v \in V(G)$ with $u \neq v$, an $(s; u, v)$-path-system of G is a subgraph H consisting of s internally disjoint (u, v)-paths. A graph is **spanning s-connected** if for any $u, v \in V(G)$ with $u \neq v$, G has a spanning $(s; u, v)$-path-system. The **spanning connectivity** $\kappa^*(G)$ of a graph G is the largest integer s such that G has a spanning $(k; u, v)$-path-system, for any integer k with $1 \leq k \leq s$, and for any $u, v \in V(G)$ with $u \neq v$. An edge counter-part of $\kappa^*(G)$, defined as the supereulerian width of a graph G, has been investigated in [2]. In [1], Catlin and Lai proved that if a graph G has 2 edge-disjoint spanning trees, and if $L(G)$ is the line graph of G, then $\kappa^*(L(G)) \geq 2$ if and only if $\kappa(L(G)) \geq 3$. In this paper, we extend this result and prove that for any integer $k \geq 2$, if G has k edge-disjoint spanning trees, then $\kappa^*(L(G)) \geq k$ if and only if $\kappa(L(G)) \geq \max\{3, k\}$.

Key words: connectivity, spanning connectivity, hamiltonian line graph, hamiltonian-connected line graph, supereulerian graphs, collapsible graphs,

References

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*Department of Mathematics, West Virginia University, Morgantown, WV 26506
†Butler University, Indianapolis, IN 46208
‡College of Mathematics and System Sciences, Xinjiang University, Urumqi, Xinjiang 830046, PRC
§Department of Mathematics, Renming University of China, Beijing 100872, P. R. China