Spanning disks in toroidal embeddings

Chris Stephens Middle Tennessee State University

Abstract

Let Φ be an embedding of a graph G in a surface S. If there exists a subset K of S bounded by a subgraph of G such that K contains all vertices of G, then K is called a spanning subset of Φ .

Examples of spanning subsets include spanning disks and spanning annuli with some number of holes (the latter are called planarizing sets in some papers). A spanning subset may provide a simpler structure, yet still contain enough information to approach certain problems about graphs embedded on surfaces. In this talk, we prove that any embedding of a 4-connected graph in the torus with representativity at least three has a "spanning disk"—i.e., a contractible disk which contains all vertices of the graph and which is bounded by a cycle of the graph. Some potential applications will be discussed. This is joint work with Xiaoya Zha.