Packing Two Graphs with Bounded Sum of Edges and Maximum Degree

Derrek Yager, University of Illinois at Urbana-Champaign

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be graphs of order n and $G_3 = (V_1 \cup V_2, E_3)$ a bipartite graph. A bijection f from V_1 onto V_2 is a *list packing* of the triple $G = (G_1, G_2, G_3)$ if $uv \in E_1$ implies $f(u)f(v) \notin E_2$ and $vf(v) \notin E_3$ for all $v \in V_1$. Also, let $D_i := \max_{v \in V_i} \max\{d_i(v), d_3(v)\}$ for $i = 1, 2$. We prove that if $\Delta_1, \Delta_2 \leq n-2$ and $\Delta_3 \leq n-1$, then there is an absolute constant C such that if $e_1 + e_2 + e_3 + D_1 + D_2 \leq 3n - C$, then G packs. In order to prove this, we also show that if $\Delta_1 \Delta_2 + \Delta_3 < \frac{n}{2}$, then G packs, an extension of a result by Sauer and Spencer. Additionally, we extend a result of Bollobás and Eldridge, proving that if $\Delta_1, \Delta_2 \leq n-2, \Delta(G_3) \leq n-1$, and $e_1 + e_2 + e_3 \leq 2n - 3$, then G packs or is one of 8 possible exceptions.