Hamilton cycles in 3-connected claw-free and net-free graphs

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For an integer $s_1, s_2, s_3 > 0$, let N_{s_1,s_2,s_3} denote the graph obtained by identifying each vertex of a K_3 with an end vertex of three disjoint paths P_{s_1+1}, P_{s_2+1}, P_{s_3+1} of length s_1, s_2, and s_3, respectively. We determine a family $\mathcal{F}$ of graphs such that, every 3-connected $(K_{1,3}, N_{s_1,s_2,1})$-free graph Γ with $s_1 + s_2 + 1 \leq 10$ is hamiltonian if and only if the closure of Γ is $L(G)$ for some graph $G \notin \mathcal{F}$. We also obtain the following results.

1. Every 3-connected $(K_{1,3}, N_{s_1,s_2,s_3})$-free graph with $s_1 + s_2 + s_3 \leq 9$ is hamiltonian.

2. If G is a 3-connected $(K_{1,3}, N_{s_1,s_2,0})$-free graph with $s_1 + s_2 \leq 9$, then G is hamiltonian if and only if the closure of G is not the line graph of a member in $\mathcal{F}$.

3. Every 3-connected $(K_{1,3}, N_{s_1,s_2,0})$-free graph with $s_1 + s_2 \leq 8$ is hamiltonian.

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