P_4-decomposition of regular graphs and multigraphs

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Let $\mu(G)$ denote the maximum edge multiplicity of a multigraph G, and let P_4 denote the path on four vertices. Heinrich, Liu and Yu, and independently Adelgren, proved that P_4 decomposes a connected 4-regular simple graph G if and only if the edge set cardinality $|E(G)|$ is divisible by 3. We show that P_4 decomposes a connected 4-regular multigraph G with $\mu(G) \leq 2$ if and only if $|E(G)|$ is divisible by 3, and no 3 vertices of G induce more than 4 edges. Both proofs rely on establishing the existence of certain triangle-free euler tours. We give some generalizations of this result for higher edge multiplicities, but also show that certain variations on the vertex degree or the multiplicities yield problems that are NP-complete.

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