On proper path-colorings in graphs

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Let G be an edge-colored connected graph. A path P is a proper path in G if no two adjacent edges of P are colored the same. If P is a proper $u - v$ path of length $d(u, v)$, then P is a proper $u - v$ geodesic. An edge coloring c is a proper-path coloring of a connected graph G if every pair u, v of distinct vertices of G are connected by a proper $u - v$ path in G and c is a strong proper coloring if every two vertices u and v are connected by a proper $u - v$ geodesic in G. These concepts are inspired by the concepts of rainbow coloring and strong rainbow coloring of a connected graph G. We investigate the relationship among these four edge colorings as well as the well-studied proper edge colorings in graphs.