

Surviving rate of planar digraphs

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Let D be a connected digraph with $n \geq 2$ vertices. Suppose that a fire breaks out at a vertex v of D . A firefighter starts to protect vertices. At each time interval, the firefighter protects k vertices not yet on fire. Afterwards, the fire spreads to all unprotected neighbors that are heads of some arcs starting from the vertices on fire. Let $\text{sn}_k(v)$ denote the maximum number of vertices in D that the firefighter can save when a fire breaks out at vertex v . The k -surviving rate $\rho_k(D)$ of D is defined as $\sum_{v \in V(D)} \text{sn}_k(v)/n^2$. We obtain the following results.

- (1) If D is a planar digraph, then $\rho_2(D) > \frac{1}{40}$;
- (2) If D is a planar digraph without 4-cycles, then $\rho_1(D) > \frac{1}{51}$.