

Reversible Peg Solitaire on Graphs

John Engbers, Marquette University

Peg solitaire is a board game that involves pegs occupying all but one of the spaces on some game board, with the goal of the game to use geometric jumps to reduce the number of pegs to one. The game has been played on various types of boards, including a cross (the English board) and a triangle (the game Eg-no-ra-moose, which is common in Cracker Barrel restaurants). Recently the game was generalized, in a combinatorial sense, to graphs. In peg solitaire on graphs, pegs are initially placed on all but one vertex of a graph G . If xyz forms a path in G and there are pegs on vertices x and y but not z , then a *jump* places a peg on z and removes the pegs from x and y . A graph is called solvable if, for some initial configuration of pegs, some sequence of jumps leaves a single peg left. Determining which graphs are solvable in peg solitaire seems to be a difficult question.

We study the game of *reversible peg solitaire*, where there are again initially pegs on all but one vertex, but now both jumps and unjumps (the reversal of a jump) are allowed. We show that in this game all non-star graphs that contain a vertex of degree at least three are solvable, and we also show that cycles and paths on n vertices, where n is divisible by 2 or 3, are solvable. Several open questions remain. This is joint work with Christopher Stocker.