Unit vector flow with rank at most 2 and nowhere-zero 3-flows

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An S^d-flow is a flow whose flow values are vectors in S^d, where S^d is the set of unit vectors on $\mathbb{R}^{d+1}$. Kamal Jain conjectured that every 2-edge-connected graph has an S^2-flow, and every 4-edge-connected graph has an S^1-flow. Thomassen pointed out that a graph has an S^1-flow if it has a nowhere zero 3-flow, but its converse is not true and gave a class of examples which admit S^1-flows but have no nowhere zero 3-flows.

The rank of a S^1-flow is defined as the rank of linear space generated by all balanced vectors $\overrightarrow{\lambda}(v) = (\lambda_1(v), \lambda_2(v), \ldots, \lambda_k(v))$ for all $v \in V(G)$, where $\lambda_i(v)$ is the difference between the number of out-edges with flow value α_i from v and the number of in-edges with the same flow value to v. In this paper, We find that the rank of Thomassen's counterexample is great than 2, and prove that G has a nowhere 3-flow if G admits an S^1-flow with rank at most 2.