The representation number of certain sparse graphs

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A representation modulo r of a (finite, simple) graph G is a labeling of the vertices of G by distinct integers from $\{0, \ldots, r-1\}$ such that two vertices are adjacent if and only if the difference of their labels is relatively prime to r; the representation number of G is the smallest integer r for which such a labeling exists. The exact value of the representation number is known only in very few cases, mostly for graphs with many edges. Moreover, the quantity does not appear to be well-behaved with respect to standard graph-theoretic operations, but instead seems to be connected to questions concerning the distribution of primes. We compute the representation number for complete binary trees and for graphs with a single edge.