CE 34500: Transportation Engineering
Homework 6

Problem 1: A +2 % grade intersects with a 1 % grade at station (535+24.25) at an elevation of 300 ft. If the design speed is 65 mph, determine:

a) The minimum length of vertical curve using the rate of vertical curvature. Then using the length found in part (a), find:
b) The stations and elevations of the BVC and EVC
c) The elevation of each 100-ft. station
d) The station and elevation of the high point

Solution:

From Table 15.4, K = 193
L = KA = 193 (2 – (–1)) = 579 ft

Station of BVC = (535+24.25) – (579 ft)/2 = 532+34.75
Station of EVC = (535+24.25) + (579 ft)/2 = 538+13.75

Elevation of BVC = 300 – (0.02)(579/2) = 294.21 ft

Elevation at any station *on the leading tangent* can be found in a similar manner. The elevation on the curve can be found by subtracting the elevation on the leading tangent by the offset, which can be found using Equation 15.12,

$$Y = \frac{A}{200L}x^2$$

Using this procedure, the following table, which tabulates the elevation at 100 ft stations on the curve, can be generated.

Station	Dist. from BVC	Tangent Elev.	Offset	Curve Elev.
532+34.75	0	294.21	0	294.21
533+00	65.25	295.52	0.11	295.41
534+00	165.25	297.52	0.71	296.81
535+00	265.25	299.52	1.82	297.70
536+00	365.25	301.52	3.46	298.06
537+00	465.25	303.52	5.61	297.91
538+00	565.25	305.52	8.28	297.24
538+13.75	579.00	305.79	8.69	297.10

The distance from the BVC to the high point can be found as:
$x_{high} = LG_1 / (G_1 - G_2) = (579)(2)/(2 - (-1)) = 386$ ft
The station of the high point is (532+34.75) + (386 ft) = 536+20.75
The difference between the elevation of the BVC and the elevation of the high point can be found as:
$y_{high} = LG_1^2 / (200(G_1 - G_2)) = (579)(2)^2 / (200)(2 - (-1)) = 3.86$ ft
Therefore, the elevation of the high point is 294.21 + 3.86 = 298.07 ft

Problem 2: Determine the minimum length of a crest vertical curve, using the minimum length based on SSD criteria if the grades are +4% and -2%. Design speed is 70 mph. State assumptions used.

Solution:

Assumptions used include: perception-reaction time is 2.5 seconds, deceleration rate is 11.2 ft/sec^2, and the case is sight distance is less than the length of the curve.

Determine required stopping sight distance using Equation 3.27:

$$SSD = 1.47ut + \frac{u^2}{30(\frac{a}{g} \pm G)} = 1.47(70)(2.5) + \frac{(70)^2}{30(\frac{11.2}{32.2} - 0.04)}$$

$SSD = 788$ ft

Since S < L, use Equation 15.5 to calculate the minimum length of the curve.

$$L_{min} = \frac{AS^2}{2158} = (6)(788)^2/2158 = 1727 \text{ ft}$$

Therefore, for the given design conditions, the minimum length of the curve is 1,727 ft.

Problem 3: Determine the minimum length of a sag vertical curve if the grades are -4% and +2%. Design speed is 70 mph. State assumptions used. Consider the following criteria: stopping sight distance, comfort, and general appearance.

Solution:

Assumptions used include: perception-reaction time is 2.5 seconds, deceleration rate is 11.2 ft/sec^2, and the case is sight distance is less than the length of the curve.

For the sight distance criterion:
Determine required stopping sight distance using Equation 3.27:

$$SSD = 1.47ut + \frac{u^2}{30(\frac{a}{g} \pm G)} = 1.47(70)(2.5) + \frac{(70)^2}{30(\frac{11.2}{32.2} - 0.04)}$$

$SSD = 788$ ft

Since S<L, use Equation 15.9 to calculate the minimum length of the curve.

$$L = \frac{AS^2}{400 + 3.5S} = \frac{(6)(788)^2}{400 + 3.5(788)} = 1179 \text{ ft}$$

For the comfort criterion, use Equation 15.10:

$$L = \frac{Au^2}{46.5} = (6)(70)^2/46.5 = 633 \text{ ft}$$

For the general appearance criterion, use Equation 15.11:

$$L = 100\,A = (100)(6) = 600 \text{ ft}$$

Therefore, for the given design conditions, the minimum length of the curve is 1,179 ft.

Problem 4: Given a sag vertical curve connecting a -1.5 % grade with a +2.5% grade on a rural arterial highway, use the rate of vertical curvature, and a design speed of 70 mph to compute the elevation of the curve at 100 ft. stations if the grades intersect at station (475+00) at an elevation of 300 ft. Identify the station and elevation of the low point.

Solution:

From Table 15.5, K = 181
L = KA = 181 (1.5 – (–2.5)) = 724 ft

Station of BVC = (475+00) – (724 ft)/2 = 471+38.00
Station of EVC = (475+00) + (724 ft)/2 = 478+62.00

Elevation of BVC = 300 + (0.015)(724/2) = 305.43 ft
Elevation at any station *on the leading tangent* can be found in a similar manner. The elevation on the curve can be found by adding the elevation on the leading tangent to the offset, which can be found using Equation 15.12,

$$Y = \frac{A}{200L}x^2$$

Using this procedure, the following table, which tabulates the elevation at 100 ft stations on the curve, can be generated.

Station	Dist. from BVC	Tangent Elev.	Offset	Curve Elev.
471+38	0	305.43	0	305.43
472+00	62	304.50	0.11	304.61
473+00	162	303.00	0.72	303.72
474+00	262	301.50	1.90	303.40
475+00	362	300.00	3.62	303.62
476+00	462	298.50	5.90	304.40
477+00	562	297.00	8.72	305.72
478+00	662	295.50	12.11	307.61
478+62	724	294.57	14.48	309.05

The distance from the BVC to the low point can be found as:
$x_{low} = LG_1 / (G_1 – G_2) = (724)(1.5)/(1.5 – (–2.5)) = 271.50$ ft
The station of the low point is (471+38) + (2+71.50) = 474+09.50
The difference between the elevation of the BVC and the elevation of the low point can be found as:
$y_{low} = LG_1^2 / (200(G_1 – G_2)) = (724)(1.5)^2 / (200)(1.5 – (–2.5)) = 2.04$ ft
Therefore, the elevation of the low point is 305.43 – 2.04 = 303.39 ft

Problem 5: A crest vertical curves connects a +4.44% grade and a -6.87% grade. The PVI is at station 43+50.00 at an elevation of 1240.00 ft. The design speed is 30 mph.

a) The length of the vertical curve using the AASHTO method ("K" factors)
b) The stations of the BVC,
c) The elevation of the BVC,
d) The stations of the EVC,
e) The elevation of the EVC,
f) The stations of the high point,
g) The elevation of the high point,
h) The elevation of station 44+23.23

Solution:

From Table 15.4, K = 19

(a) L = KA = (19)|–4.44-6.87| = (19)(11.31) = 214.89 ft

(b) Station of BVC = (43+50) – (214.89 ft)/2 = 42+42.55

(c) Elevation of BVC = 1240.00 – (0.0444)(214.89 ft/2) = 1235.23 ft

(d) Station of EVC = (43+50) + (214.89 ft)/2 = 44+57.45

(e) Elevation of EVC = 1240.00 – (0.0687)(214.89 ft/2) = 1232.62 ft

(f) Station of the high point = (42+42.55) + (214.90)(4.44)/(11.31) = 43+26.91

(g) Elevation of the high point $= 1235.23 + \dfrac{(214.90)(4.44)^2}{(200)(11.31)} = 1237.10$ ft

(h) Elevation of station 44+23.23 =

$$1235.23 + (0.0444)(180.68) - \frac{(11.31)(180.68)^2}{(200)(214.90)} = 1234.66 \text{ ft}$$

Problem 6: A horizontal curve is to be designed for a two-lane road in mountainous terrain. The following data are known: intersection angle 40 degrees, tangent length 436.76 ft., and station of $PI: 2700 + 10.65, fs = 0.12, e = 0.08.$ Determine:

a) Design speed
b) Station of the PC and
c) PT
d) Deflection angle and chord length to the first 100 ft. station

Solution:

(a) From the given horizontal curve data, the radius can be calculated, from which design speed for the curve can be derived.

The radius can be found by rearranging Equation 15.22,

$R = T / (\tan \Delta/2) = 436.76 / \tan (40°/2) = 436.76 / 0.3640 = 1200$ ft

The design speed can then be found:

$R = u^2 / [15(e + f_s)]$

$(1200) = u^2 / [\ 15\ (0.08+0.12)\ ]$

$u^2 = 3600$

$u = 60$ mi/h

(b) Station of the PC can be found by subtracting the tangent length from the station of the PI.

PC = (2700+10.65) – (4+36.76) = 2695+73.89

(c) The length of the curve can be found using Equation 15.26,

$L = \pi R\Delta/180 = (3.1415926)(1200)(40)/180 = 837.76$ ft

Station of the PT can be found by adding the length of the curve to the station of the PC.

PT = (2695+73.89) + (8+37.76) = 2704+11.65

(d) Deflection angle and chord length to the first full station

To find the deflection angle to the first full station, use Equation 15.27,

$\delta_1/2 = 180\ l_1 / 2\pi R = (180)(100–73.89)/(2)(3.1415926)(1200) = 0.6233°$

The chord to the first full station can be found using Equation 15.28,

$C_1 = 2R \sin (\delta_1/2) = (2)(1200) \sin (1.24666/2) = 26.11$ ft

Problem 7: Given a sample circular curve with the following properties: $D = 11°$, bearing on incoming (back) tangent is $N\,89°27'25''\,E$, bearing on outgoing (forward) tangent is $S\,60°10'05''E$. The station of the PI = 22+69.77. Determine

a) The intersection angle
b) Radius
c) Tangent
d) The external distance
e) The middle ordinate
f) The long chord
g) The length of the curve
h) Station of the PC
i) and PT

Solution:

(a) $\Delta = 180° – (89.45694° + 60.16806°) = 30.375°$

(b) $R = 5729.58 / 11 = 520.87$ ft

(c) $T = R \tan(\Delta/2) = (520.87) \tan(30.375/2) = 141.39$ ft

(d) $E = 520.87[\frac{1}{\cos(30.375/2)} - 1] = 18.85$ ft

(e) $M = 520.87 [1 - \cos(30.375/2)] = 18.19$ ft

(f) $C = 2(520.87) \sin(30.375/2) = 272.91$ ft

(g) $L = (30.375)(520.87)\pi/180 = 276.14$ ft

(h) Station PC = (22+69.77) – 141.39 ft = 21+28.38

(i) Station PT = (21+28.38) + 276.14 ft = 24+04.52