Chapter 2 – Data Representation

CS 271 Computer Architecture

Purdue University Fort Wayne

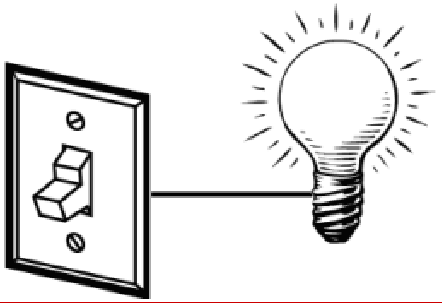
Chapter 2 Objectives

- Understand the *fundamentals* of numerical data representation and *manipulation* in computer systems.
- Master the skill of *converting* between different numeric-radix systems.
- Understand how *errors* can occur in computations because of overflow and truncation.
- Understand the *fundamental* concepts of floating-point representation.
- Gain familiarity with the most *popular character* codes.

Outline

- Converting between different numeric-radix systems
- Binary addition and subtraction
- Two's complement representation
- Floating-point representation
- Characters in computer

2.1 Introduction

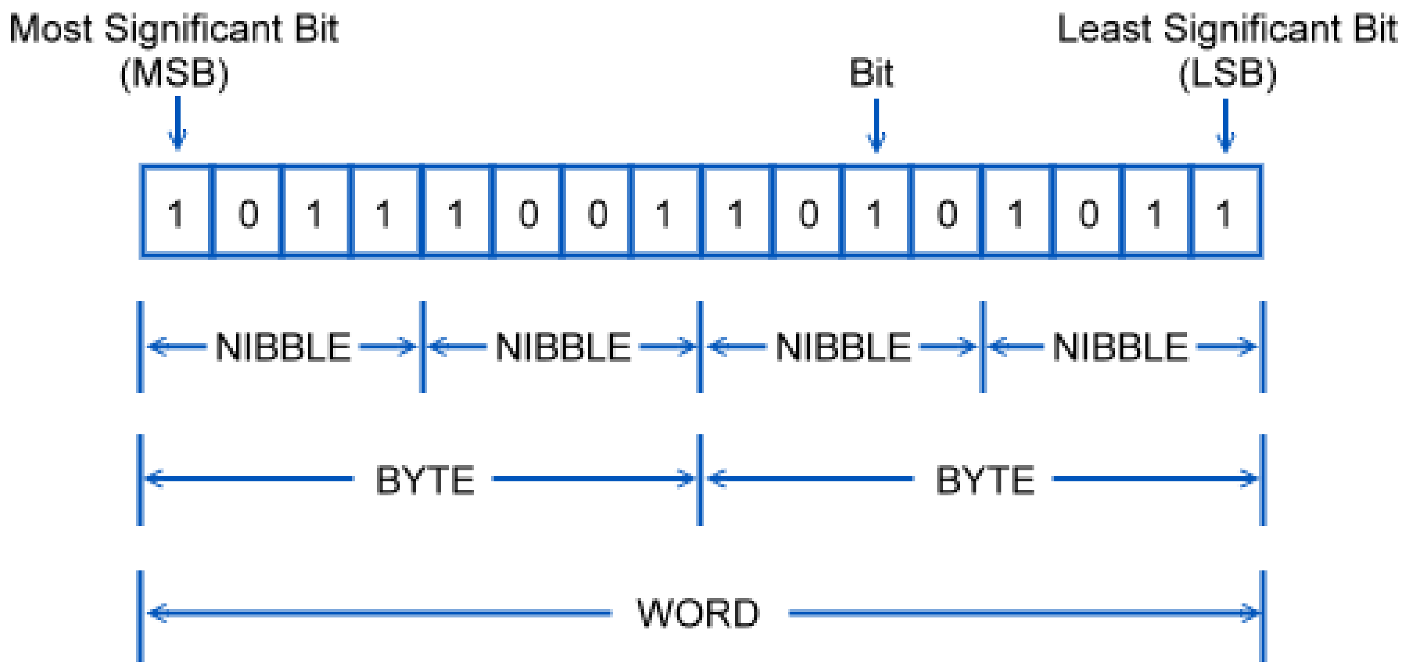


- A *bit* is the most basic unit of information in a computer.
  - It is a state of either "on" or "off", "high" or "low" voltage in a digital circuit.
  - In a computer, a bit could be either "1" or "0".
- A *byte* is a group of 8 bits.
  - a byte is the *smallest* possible unit of storage in computer systems
- A group of 4 bits is called a *nibble*.
  - A byte consists of 2 nibbles:
    - The "*high-order*" nibble and the "*low-order*" nibble.

2.1 Introduction

- A *word* is a contiguous group of bytes.
  - Words can be any number of bits or bytes.
  - According to different computer systems, the size of word could be *2 bytes* (16 bits), *4 bytes* (32 bits), or *8 bytes* (64 bits) bits.

2.1 Introduction



Byte or Word Addressable

- A computer allows either a byte or a word to be *addressable*
  - *Addressable*: a particular unit of storage can be retrieved by CPU, according to its location in memory.
  - A byte is the *smallest* possible *addressable* unit of storage in a *byte-addressable* computer
  - A word is the smallest addressable unit of storage in a *word-addressable* computer

2.2 Positional Numbering Systems

- Bytes store numbers use the position of each bit to represent a *power of 2 (radix of 2)*.
  - The binary system is also called the *base-2* system.
  - Our decimal system is the *base-10* system, because the position of each number represents a *power of 10 (radix of 2)*.
- When the radix of a number is other than 10, the base is denoted as a subscript.
  - Sometimes, the subscript 10 is added for emphasis

2.2 Positional Numbering Systems

- Let’s first look at numbers in base-10 number system
- The decimal number 947_{10} (**base-10**) is:

  $947_{10} = 9 \times 10^{2} + 4 \times 10^{1} + 7 \times 10^{0}$

- The decimal number 5836.47_{10} (**base-10**) is:

  $5836.47_{10} = 5 \times 10^{3} + 8 \times 10^{2} + 3 \times 10^{1} + 6 \times 10^{0} + 4 \times 10^{-1} + 7 \times 10^{-2}$

2.2 Positional Numbering Systems

- Then, look at numbers in base-2 number system
- The binary number 11001_2 (**base-2**) is:

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 16 + 8 + 0 + 0 + 1 = 25_{10}$$

- $11001_2 = 25_{10}$

Practice

- $(01111101)_2 = ?$

- $(123)_8 = ?$

- $(123)_3 = ?$

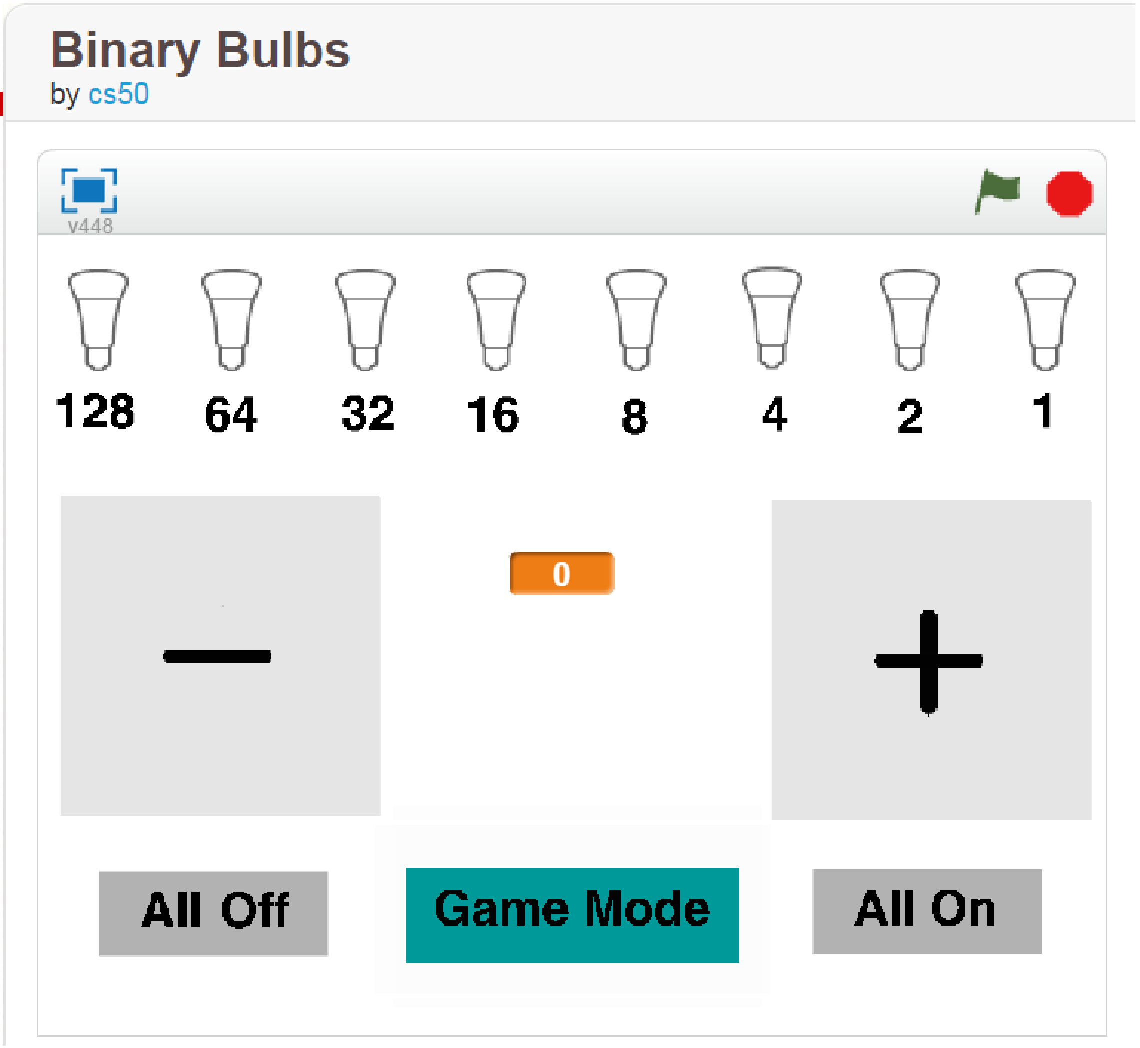
Any problem?

Why Do I Have To Learn And Convert Binary Numbers?

- Because binary numbers are the basis for all data representation in computer systems
  - It is important that you become proficient with binary system.
- Your knowledge of the binary numbering system will help you understand the operations of all computer components
  - As well as the instruction set of computer.

2.3 Converting Between Bases

- How can any integer (base-10 number) be converted into any radix system?
- There are two methods of conversion:
  - The ***Subtraction*** ***(-)*** method, and
  - The ***Division*** ***(/)*** remainder method.
- Let's use the subtraction method to convert $\mathbf{190_{10}}$ to $\mathbf{X_3}$.



Binary Bulbs
by cs50

v448

128 64 32 16 8 4 2 1

0

All Off
Game Mode
All On

- https://scratch.mit.edu/projects/26329434/

2.3 Converting Between Bases: *Subtraction*

- **Converting the decimal number 190_{10} to X_3.**
  - Let's try different integers $i=3^n$
  - $3^5 = 243 > 190$
  - The largest power of 3 that we need is $3^4 = 81 < 190$, and $81 \times 2 = 162$.
  - Write down the 2 and subtract 162 from 190, get the remainder 28.

$$\begin{array}{r} 190 \\ -\ 162 \\ \hline 28 \end{array} \quad = 3^4 \times \boxed{2}$$

2.3 Converting Between Bases: ***Subtraction***

- **Converting 190_{10} to X_3...**
  - The next power of 3 is 3^{3} = 27 < 28, so we subtract 27 and write down the numeral 1 as our result.
  - The next power of 3 is 3^{2} = 9 > 1, too large, so we skip 3^{2}

$$\begin{array}{rrcl}
 & 190 & & \\
- & 162 & = & 3^4 \times 2 \\
\hline
 & 28 & & \\
- & 27 & = & 3^3 \times 1 \\
\hline
 & 1 & & \\
- & 0 & = & 3^2 \times 0 \\
\hline
 & 1 & &
\end{array}$$

2.3 Converting Between Bases: *Subtraction*

- **Converting 190_{10} to X_3...**
  - $3^1 = 3 > 1$. too large, so we skip 3^1
  - The last power of 3 = 3^0 = 1, is our last choice, and it gives us a difference of 0.
  - Our result, reading from top to bottom is:

    $190_{10} = 21001_3$

$$\begin{array}{rr|l}
 & 190 & \\
- & 162 & = 3^4 \times 2 \\
\hline
 & 28 & \\
- & 27 & = 3^3 \times 1 \\
\hline
 & 1 & \\
- & 0 & = 3^2 \times 0 \\
\hline
 & 1 & \\
- & 0 & = 3^1 \times 0 \\
\hline
 & 1 & \\
- & 1 & = 3^0 \times 1 \\
\hline
 & \boxed{0} &
\end{array}$$

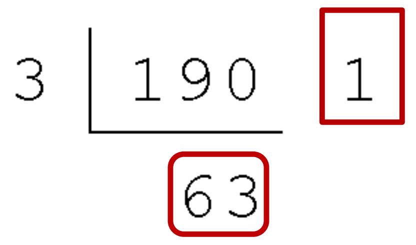
The subtraction method is more intuitive, but cumbersome.

2.3 Converting Between Bases: *Division*

- Let’s try another method of conversion: **Division**.
- The idea is that:
  - Successive division by a base is equivalent to successive subtraction by powers of the base!
- Let’s use the division method to convert $\mathbf{190_{10}}$ to $\mathbf{X_3}$, again.

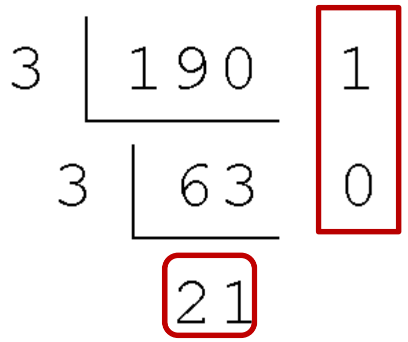
2.3 Converting Between Bases

- **Converting 190_{10} to base 3...**
  - First we take the number that we wish to convert and divide it by the radix in which we would like to convert to.
  - In this case, 190/3 → the quotient is 63, and the remainder is 1.



2.3 Converting Between Bases

- **Converting 190_{10} to base 3...**
  - 63 is evenly divisible by 3.
  - The quotient is 21, the remainder is 0.

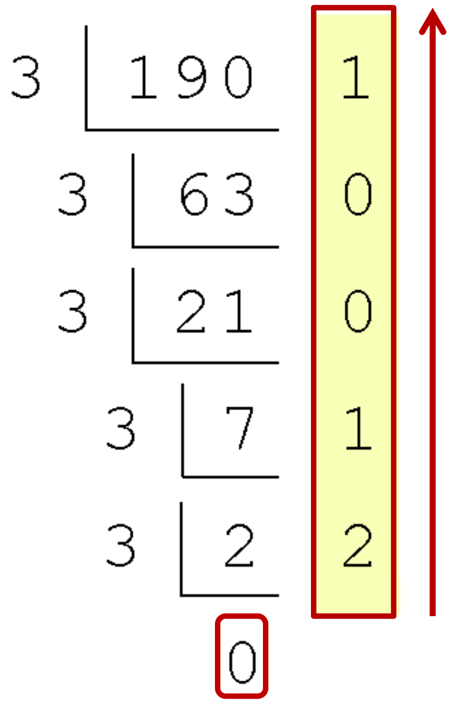


2.3 Converting Between Bases

- **Converting 190_{10} to base 3...**
  - Continue in this way until the quotient is 0.
  - In the final calculation, we note that 3 divides 2 zero times with a remainder of 2.
  - Our result, reading from bottom to top is:

    $190_{10} = 21001_3$

It is mechanical but easier!



Exercise

- $458_{10}=$_______________$_2$
- $652_{10}=$_______________$_2$
- $458_{10}=$_______________$_3$
- $652_{10}=$_______________$_5$

- Once you get the result, please verify your result by converting back!
- Don’t user calculator!

2.3 Converting Fractional Numbers

- Fractional numbers can be approximated in all base systems, too.
- Unlike integer values, fractional numbers do not necessarily have exact representations under all radices.
  - The quantity ½ is exactly representable in the binary and decimal systems, but is not in the base 3 numbering system.

2.3 Converting Fractional Numbers

- Fractional decimal numbers have non-zero digits on the right of the decimal point.
  - Fractional values of other radix systems have nonzero digits on the right of the *radix point*.
- Numerals on the right of a radix point represent negative powers of the radix. For example

$$
\begin{aligned}
0.47_{10} &= 4 \times 10^{-1} + 7 \times 10^{-2} \\
0.11_{2} &= 1 \times 2^{-1} + 1 \times 2^{-2} \\
&= \tfrac{1}{2} + \tfrac{1}{4} \\
&= 0.5 + 0.25 \\
&= 0.75_{10}
\end{aligned}
$$

2.3 Converting Fractional Numbers

- Like the integer conversions, you can use either of the following two methods:
  - The ***Subtraction*** ***(-)*** method, or
  - The ***multiplication*** **(*x*)** method.
- The subtraction method for fractions is same as the method for integer
  - Subtract *negative powers of the radix*.
- Always start with the *largest value* --- first, $\boldsymbol{n}^{-1}$, where $\boldsymbol{n}$ is the radix.

2.3 Converting Fractional Numbers

- **Using the *subtraction* method to convert the decimal 0.8125_{10} to X_2.**
  - Our result, reading from top to bottom is:

$0.8125_{10} = 0.1101_2$

  - Subtraction stops when the remainder becomes 0
  - Of course, this method works with any base, not just binary.

0.125

$$\begin{array}{r l}
0.8125 & \\
-\ 0.5000 & = 2^{-1} \times 1 \\ \hline
0.3125 & \\
-\ 0.2500 & = 2^{-2} \times 1 \\ \hline
0.0625 & \\
-\ 0 & = \boxed{2^{-3}} \times 0 \\ \hline
0.0625 & \\
-\ 0.0625 & = 2^{-4} \times 1 \\ \hline
0 &
\end{array}$$

2.3 Converting Fractional Numbers

- **Using the *multiplication* method to convert the decimal 0.8125_{10} to X_2, we multiply by the radix 2.**
  - The first product carries into the units place.

$$\begin{array}{r} .8125 \\ \times \quad 2 \\ \hline \boxed{1}.6250 \end{array}$$

2.3 Converting Fractional Numbers

- **Converting 0.8125_{10} to X_2. .**
  - Ignoring the value in the units place at each step, continue multiplying each fractional part by the radix.

$$\begin{array}{r} .8125 \\ \times \quad 2 \\ \hline 1.6250 \end{array}$$

$$\begin{array}{r} .6250 \\ \times \quad 2 \\ \hline 1.2500 \end{array}$$

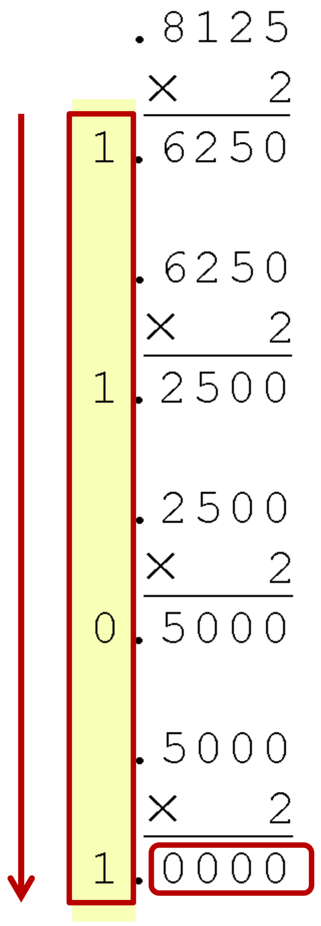
$$\begin{array}{r} .2500 \\ \times \quad 2 \\ \hline 0.5000 \end{array}$$

2.3 Converting Fractional Numbers

- **Converting 0.8125_{10} to X_2 . .**
  - You are finished when the product is 0, or until you have reached the desired number of binary places.
  - Our result, reading from top to bottom is:

    $$0.8125_{10} = 0.1101_2$$

  - Multiplication stops when the fractional part becomes 0
  - This method also works with any base. Just use *the target radix* as the multiplier.

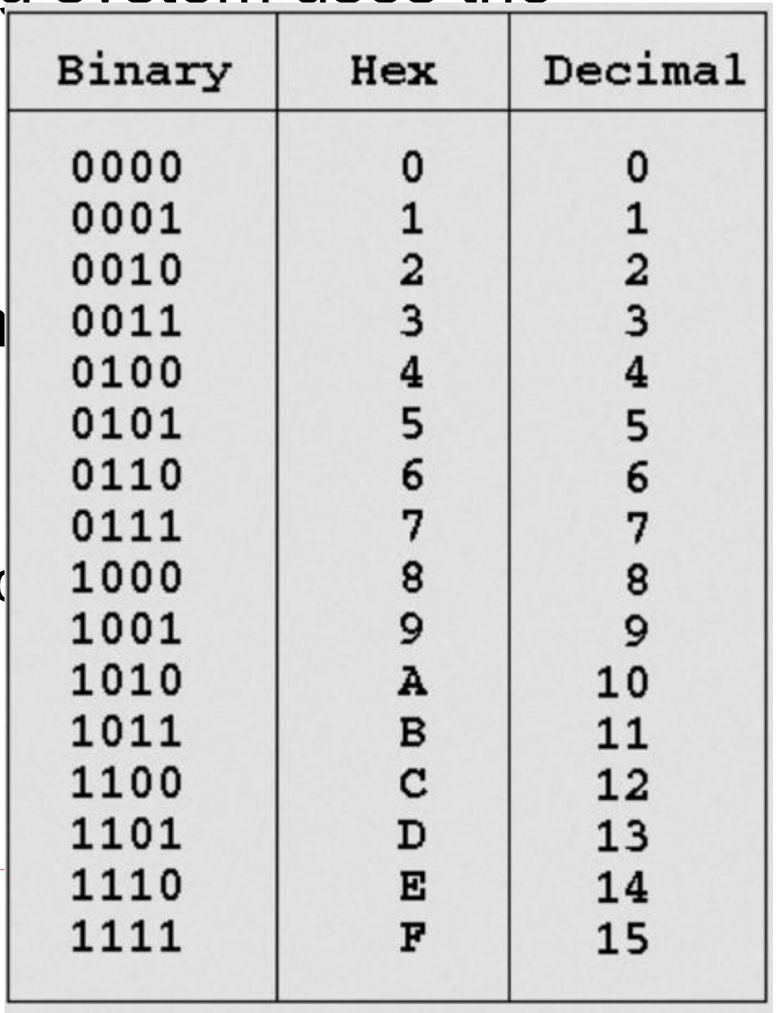


2.3 Binary and Hexadecimal Number

- Binary numbering (base 2) system is the most important radix system in computers.
- But, it is difficult to read long binary strings
  - For example: $11010100011011_2 = 13595_{10}$
- For compactness, binary numbers are usually expressed as ***hexadecimal*** (***base-16***) numbers.

2.3 Converting Between Bases

- The ***hexadecimal*** numbering system uses the numerals 0,.. ,9, A,…,F
  - $12_{10} = C_{16}$
  - $26_{10} = 1A_{16}$
- It is easy to convert between 2, because 16 = 2^4.
- Thus, to convert from binary
  - Group the binary digits into gr nibble.

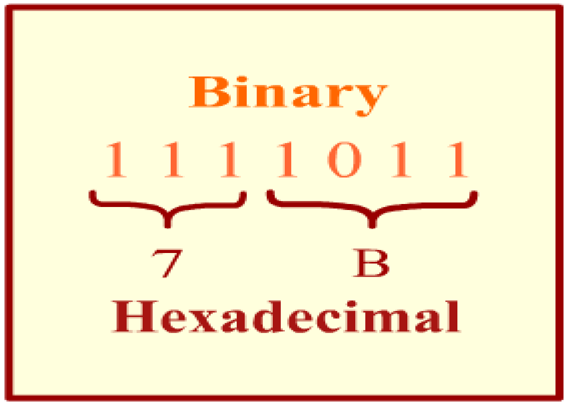


Binary	Hex	Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

Converting Binary to Hexadecimal

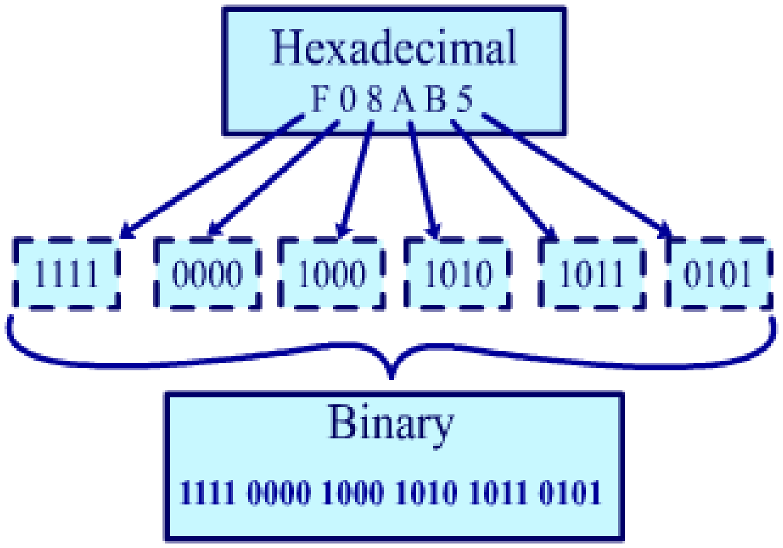
- Each hexadecimal digit corresponds to 4 binary bits.
- Example: Translate the binary integer 000101101010011110010100 to hexadecimal

1	6	A	7	9	4
0001	0110	1010	0111	1001	0100



Converting Hexadecimal to Binary

- Each hexadecimal digit can be converted to its 4-bit binary number to form the binary equivalent.



2.3 Converting Between Bases

- Using groups of hextets, the binary number 13595_{10} (= 11010100011011_2) in hexadecimal is:

If the number of bits

- Octal (base 8) values are derived from binary by using groups of three bits ($8 = 2^3$):

Octal was useful when a computer used six-bit words.

Conversion between bases 2 and 2^n

- Convert from base 2 to base 16

$$\underline{1\ 0}\ \underline{1\ 1\ 1\ 0}\ \underline{0\ 1\ 1\ 0}\ \underline{1\ 0\ 1\ 0}_2 = \qquad\qquad {}_{16}$$

- Convert from base 2 to base 8

$$1\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 0_2 = \qquad\qquad {}_{8}$$

Converting Hexadecimal to Decimal

- Multiply each digit by its corresponding power of 16:

  $\text{Decimal} = (d_3 \times 16^3) + (d_2 \times 16^2) + (d_1 \times 16^1) + (d_0 \times 16^0)$

  d_i = hexadecimal digit at the *i*th position

- Examples:
  - $1234_{16} = (1 \times 16^3) + (2 \times 16^2) + (3 \times 16^1) + (4 \times 16^0) = 4660_{10}$
  - $3BA4_{16} = (3 \times 16^3) + (11 * 16^2) + (10 \times 16^1) + (4 \times 16^0) =15268_{10}$

Exercise

- $58_{16} = \underline{\hspace{4cm}}_{10}$
- $152_{8} = \underline{\hspace{4cm}}_{10}$
- $56_{7} = \underline{\hspace{4cm}}_{10}$
- $52_{11} = \underline{\hspace{4cm}}_{10}$

- Once you get the result, please verify your result by converting back!
- Don’t user calculator!

Conversion between bases 2^m and 2^n

- Convert from base 16 to base 8
- You can use a intermediate radix number
- For example
  - Base 16 to Base 2 (binary)
  - Base 2 (binary) to Base 8

$$
\begin{aligned}
A9DB3_{16} &= 1010\ 1001\ 1101\ \ 1011\ \ 0011_{2} \\
&= 10\ 101\ 001\ 110\ 110\ 110\ 011_{2} \\
&= 2516663_{8}
\end{aligned}
$$

EXERCISES

- 176_{10} = _______________$_{16}$
- 55801_{10} =_______________$_{8}$
- $A6_{16}$ = _______________$_{13}$
- 55_{8} =_______________$_{16}$

Conversion between bases 2 and base N

- **You must familiar with** the table on right
- **You must familiar with** the following conversion:
  - Converting from `bases 2` to `base 2^n`
  - Converting between `base 2^m` and `base 2^n`
  - Converting from `bases 2` to `base 10`
  - Converting from `bases 10` (decimal) to `base 16` (hex)

hex	dec	binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Outline

- Converting between different numeric-radix systems
- **Binary addition and subtraction**
- Two's complement representation
- Floating-point representation
- Characters in computer

Binary addition

- When the sum exceeds 1, carry a 1 over to the next-more-significant column (addition rules)
  - 0 + 0 = 0 carry 0
  - 0 + 1 = 1 carry 0
  - 1 + 0 = 1 carry 0
  - 1 + 1 = 0 carry 1

Binary subtraction

- Subtraction rules
  - 0 - 0 = 0 borrow 0
  - 0 - 1 = 1 borrow 1
  - 1 - 0 = 1 borrow 0
  - 1 - 1 = 0 borrow 0

Unsigned number: Addition and subtraction

- Exercise: Use unsigned binary to compute
  - $100_{10}+10_{10}$
  - $100_{10}-10_{10}$

- Use 8-bit unsigned numbers to calculate $100_{10} + 100_{10} + 100_{10}$ using binary addition

Unsigned number: Overflow

- Possible solution:
  - If data is stored in register, you should use longer register, which can hold more bits
  - In this case, you need a register, which has at least two bytes to hold the result

Outline

- ☐ Converting between different numeric-radix systems
- ☐ Binary addition and subtraction
- ☐ **Two's complement representation**
- ☐ Floating-point representation
- ☐ Characters in computer

2.4 Signed Integer Representation

- So far, we have presented the conversions only involve ***unsigned numbers*** → all positive or 0
- To represent signed integers, computer systems user the ***high-order bit*** to indicate the sign of a number.
  - The high-order bit is the **leftmost bit**, which also called the "***Most Significant Bit***" (MSB).
    - 0 → a positive number or 0;
    - 1 → a negative number or 0.
- The remaining bits contain the value of the number

2.4 Signed Integer Representation

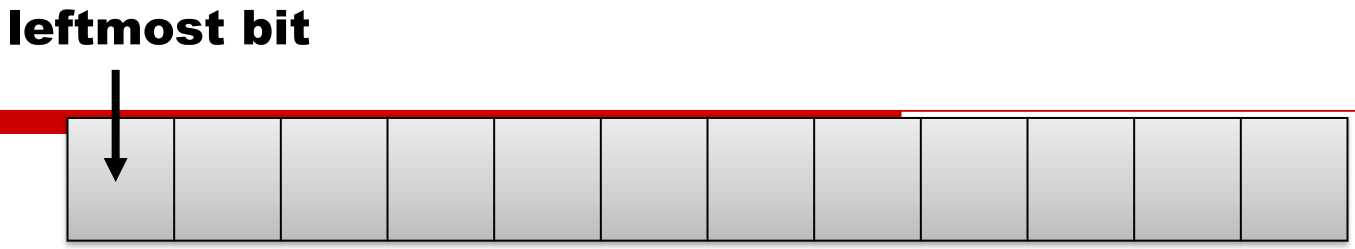
- In a byte, ***signed integer*** representation
  - 7 bits to represent *the value* of the number
  - 1 sign bit.
- There are three ways, where signed binary integers may be expressed:
  - Signed magnitude
  - One’s complement
  - **Two’s complement**

Two's Complement Representation

- ❖ Positive numbers
  - ✧ Signed value = Unsigned value
- ❖ Negative numbers
  - ✧ Signed value = Unsigned value - 2^n
  - ✧ n = number of bits

8-bit Binary value	Unsigned value	Signed value
00000000	0	0
00000001	1	+1
00000010	2	+2
. . .	. . .	. . .
01111110	126	+126
01111111	127	+127
10000000	128	-128
10000001	129	-127
. . .	. . .	. . .
11111110	254	-2
11111111	255	-1

N-bit two's complement



- Positive integer
  - Set leftmost bit to **0**
  - Express magnitude in binary in the rightmost n-1 bits
- Negative integer
  - Represent the magnitude (positive) as above
  - Then ***negate*** (***complement***) the result (see next slide), and add 1
- The leftmost bit is *the sign bit*
  - **0** for positive
  - **1** for negative

Negative Integer Representation

- 2's compliment for a negative number *-x*
  1. Represent the positive number *x* in binary
  2. *Negate* all bits
  3. *Add 1* to the result

```
Let n = 6 bits

Represent magnitude  +14₁₀ = 001110
Complement each bit          110001
Add 1                      +      1
                             110010

Result  -14₁₀ = 110010₂
```

```
Check by negating the result

Start with result    -14₁₀ = 110010
Complement each bit          001101
Add 1                      +      1
                             001110

As expected, we get  +14₁₀ = 001110₂
```

Another Example

- Represent -36 in 2’s complement format

starting value	`00100100 = +36`
step1: reverse the bits (1's complement)	`11011011`
step 2: add 1 to the value from step 1	`+        1`
sum = 2's complement representation	`11011100 = -36`

- Verification:

Sum of an integer and its 2's complement must be zero:

00100100 + 11011100 = 00000000 (8-bit sum) $\Rightarrow$ Ignore Carry

Addition and subtraction

- Addition of two's complement numbers
  - *Add all n bits* using binary arithmetic
  - *Throw away any carry* from the leftmost bit position
  - Do this whether the signs are the same or different
- For example: X-Y
  - First, negate Y. Then, add to X
  - Thus, X-Y= X + (-Y)

Examples of addition

Let n = 6 bits

Add 5 and 6 to obtain 11

$$\begin{array}{rcl} +5_{10} & = & 000101 \\ \underline{+6_{10}} & = & \underline{000110} \\ +11_{10} & = & 001011 \end{array}$$

Let n = 6 bits

-14 + 9 = –5

$$\begin{array}{rcl} -14_{10} & = & 110010 \\ \underline{+9_{10}} & = & \underline{001001} \\ -5_{10} & = & 111011 \end{array}$$

Check magnitude of -5_{10}

$$\begin{array}{lr} \text{Negate } -5_{10} = & 111011 \\ \text{Complement} & 000100 \\ \text{Add 1} & \underline{+ \quad 1} \\ \text{Magnitude: } +5 = & 000101 \end{array}$$

OK

Let n = 6 bits

-14 - 9 = -23

$$\begin{array}{rcl} -14_{10} & = & 110010 \\ \underline{-9_{10}} & = & \underline{110111} \\ -23_{10} & = & 101001 \end{array}$$

Check magnitude of -23_{10}

$$\begin{array}{lr} \text{Negate } -23_{10} = & 101001 \\ \text{Complement} & 010110 \\ \text{Add 1} & \underline{+ \quad 1} \\ \text{Magnitude: } +23 = & 010111 \end{array}$$

OK

Verification

Wrap Up

- 2’s complement:
  - Positive integer → Same
  - Negative integer → Complement all bits and add 1
- Use two’s complement to compute
  - $100_{10}+10_{10}$
  - $-100_{10}+10_{10}$
  - $100_{10}-10_{10}$
  - $-100_{10}-10_{10}$

Please first convert decimal to binary

Overflow detection

- X, Y and Z are N-bit 2's-complement numbers and $Z_{2c}=X_{2c}+Y_{2c}$
- Overflow occurs if $X_{2c}+Y_{2c}$ exceeds the maximum value represented by N-bits.
- If the signs of X and Y are different, no overflow detected for $Z_{2c}=X_{2c}+Y_{2c}$.
- In case the signs of X and Y are the same, if the sign of $X_{2c}+Y_{2c}$ is opposite, overflow detected.
  - Case 1: X, Y positive, Z sign bit ='1'
  - Case 2: X, Y negative, Z sign bit ='0'

Example

- $X_{2c}=(01111010)_{2c}$, $Y_{2c}=(10001010)_{2c}$, $X_{2c}+Y_{2c}=(00000100)_{2c}$ No overflow

$$\begin{array}{ccccccccc}
 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\
 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}$$

Carry-out 1 ignored

Example

- $X_{2c}=(11111010)_{2c}$, $Y_{2c}=(10001010)_{2c}$,

  $X_{2c}+Y_{2c}=(10000100)_{2c}$ No overflow

$$\begin{array}{ccccccccc} & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Sign = 1

Carry-out 1 ignored

Example

- $X_{2c}=(10011010)_{2c}$, $Y_{2c}=(10001010)_{2c}$, $X_{2c}+Y_{2c}=(00100100)_{2c}$ Overflow detected

$$
\begin{array}{ccccccccc}
 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
\hline
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

Sign =0

Carry-out 1

Example

- $X_{2c}=(01111010)_{2c}$, $Y_{2c}=(00001010)_{2c}$, $X_{2c}+Y_{2c}=(10000100)_{2c}$ Overflow detected

$$\begin{array}{cccccccc}
0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
\hline
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}$$

Sign=1 negative

Example

- $X_{2c}=(00111010)_{2c}$, $Y_{2c}=(00001010)_{2c}$, $X_{2c}+Y_{2c}=(01000100)_{2c}$ No overflow

$$\begin{array}{cccccccc}
0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}$$

Sign=0 positive

Programming Example

```
#include <stdio.h>

int main()
{
    int a = 32767;
    short b;

    printf ("size of int = %ld, size of short = %ld\n", sizeof(int), sizeof(short));

    b = (short)a;
    printf ("a = %d, b = %d\n", a, b);

    a ++;
    b = (short)a;
    printf ("a = %d, b = %d\n", a, b);

    return 0;
}
```

A Production Issue (C Language)

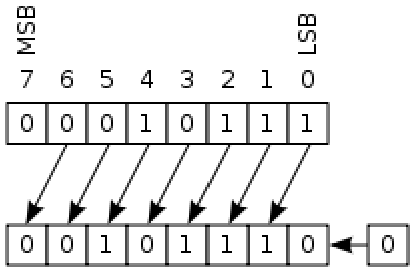
```
void do_something(short argu)
{
        ……
}

int main()
{
        int db_table_key;
        ……..
        do_something(db_table_key);
        …….
}
```

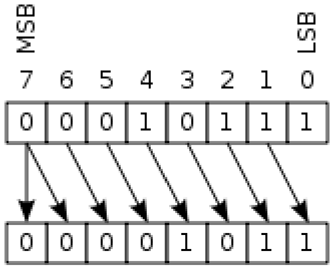
2.4 Signed Integer Representation

- In binary system, ***Multiplication/Division by 2*** very easily using an *arithmetic shift* operation
- A *left arithmetic shift* inserts a 0 in for the rightmost bit and shifts everything else left one bit; in effect, it multiplies by 2
- A *right arithmetic shift* shifts everything one bit to the right, but copies the sign bit; it divides by 2
- Let’s look at the following examples…

Bit Shifting (Arithmetic & Logical shift)



Left arithmetic shift



Right arithmetic shift

```
00010111 (decimal +23) LEFT-SHIFT
= 00101110 (decimal +46)
```

```
10010111 (decimal −105) RIGHT-SHIFT
= 11001011 (decimal −53)
```

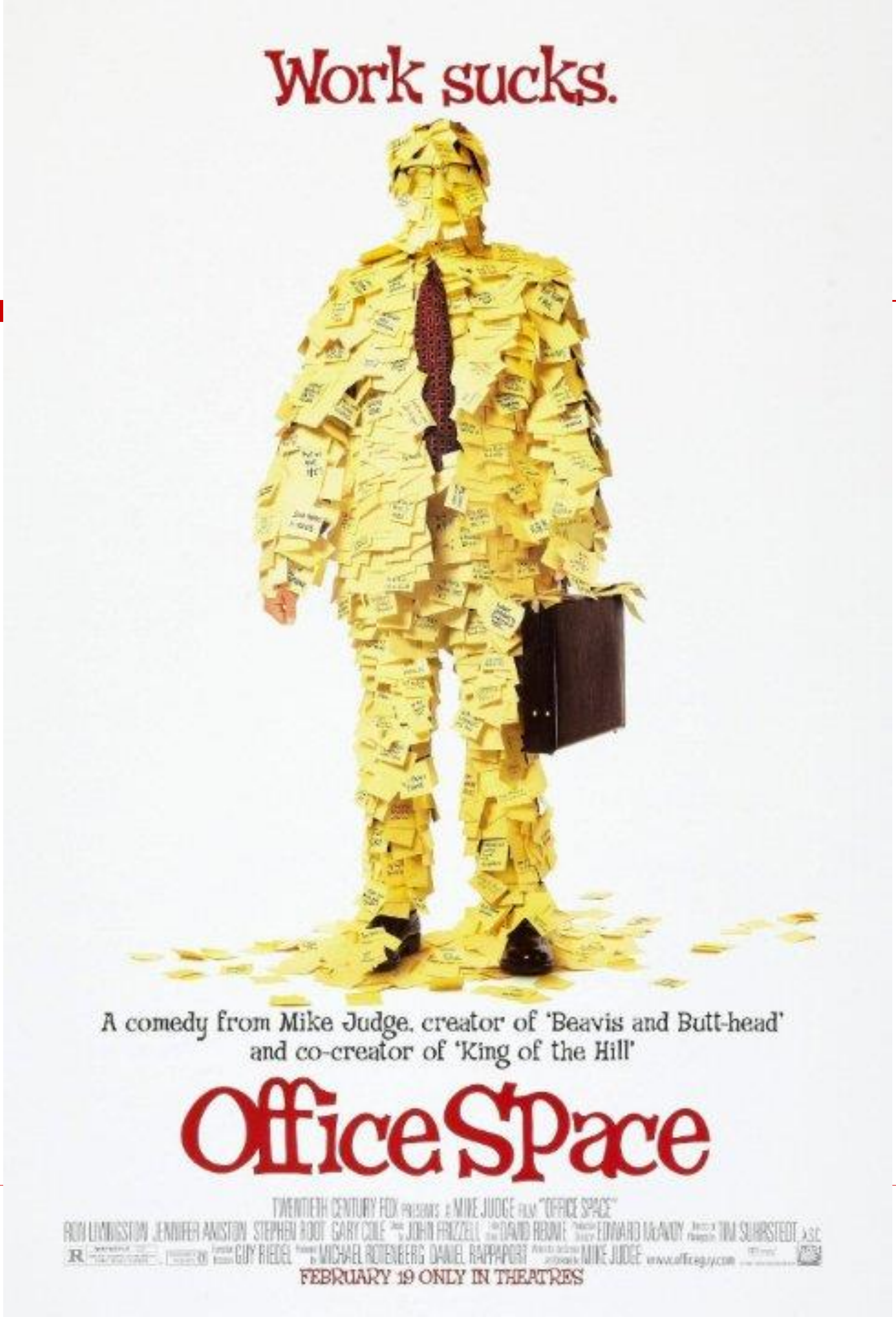
- To multiply 23 by 4, simply left-shift *twice*
- To divide 105 by 4, simply right-shift *twice*

Outline

- Converting between different numeric-radix systems
- Binary addition and subtraction
- Two’s complement representation
- **Floating-point representation**
- Characters in computer

2.5 Floating-Point Representation

- The two’s complement representation that we have just presented deals with signed integer values only.
- Without modification, these formats are not useful in scientific or business applications that deal with real number values.
- Floating-point representation solves this problem.


Work sucks.
A comedy from Mike Judge, creator of 'Beavis and Butt-head' and co-creator of 'King of the Hill'
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2.5 Floating-Point Representation

- If we are clever programmers, we can perform floating-point calculations using any integer format.
- This is called *floating-point emulation*, because floating point values aren't stored as such; we just create programs that make it seem as if floating-point values are being used.
- Most of today's computers are equipped with specialized hardware that performs floating-point arithmetic with no special programming required.

2.5 Floating-Point Representation

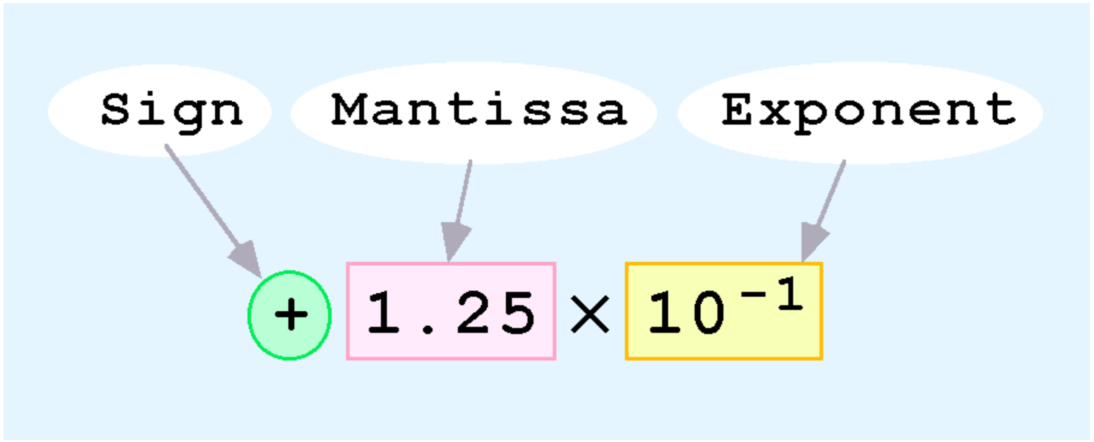
- Floating-point numbers allow an arbitrary number of decimal places to the right of the decimal point.
  - For example: $0.5 \times 0.25 = 0.125$
- They are often expressed in scientific notation.
  - For example:

    $0.125 = 1.25 \times 10^{-1}$

    $5{,}000{,}000 = 5.0 \times 10^{6}$

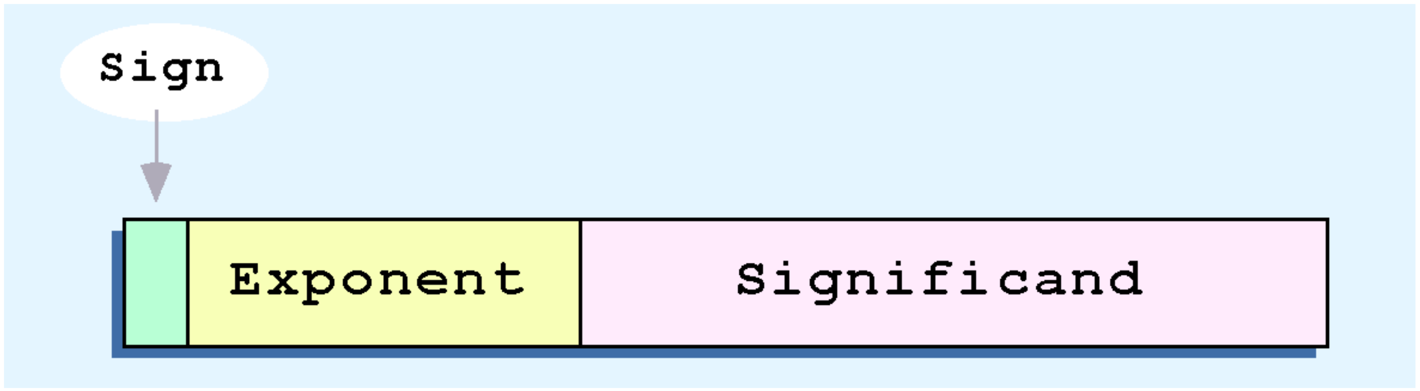
2.5 Floating-Point Representation

- Computers use a form of scientific notation for floating-point representation
- Numbers written in scientific notation have three components:



2.5 Floating-Point Representation

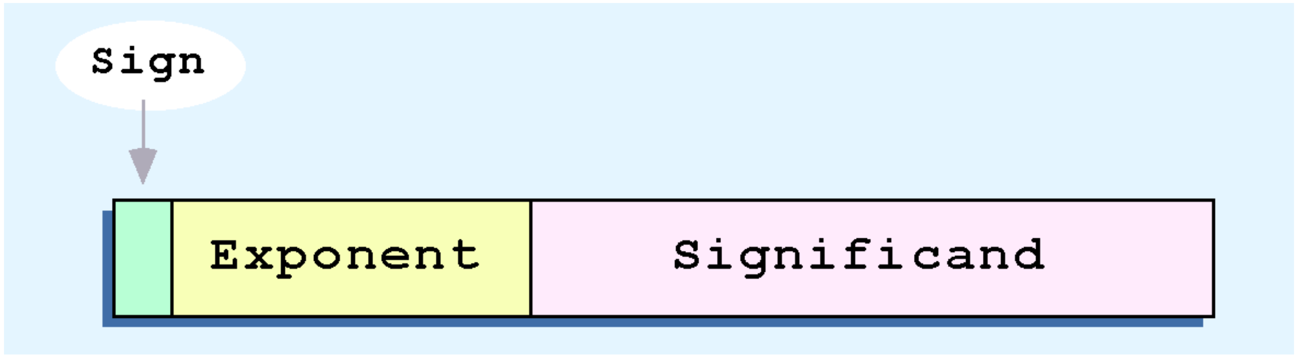
- Computer representation of a floating-point number consists of three fixed-size fields:



- This is the standard arrangement of these fields.

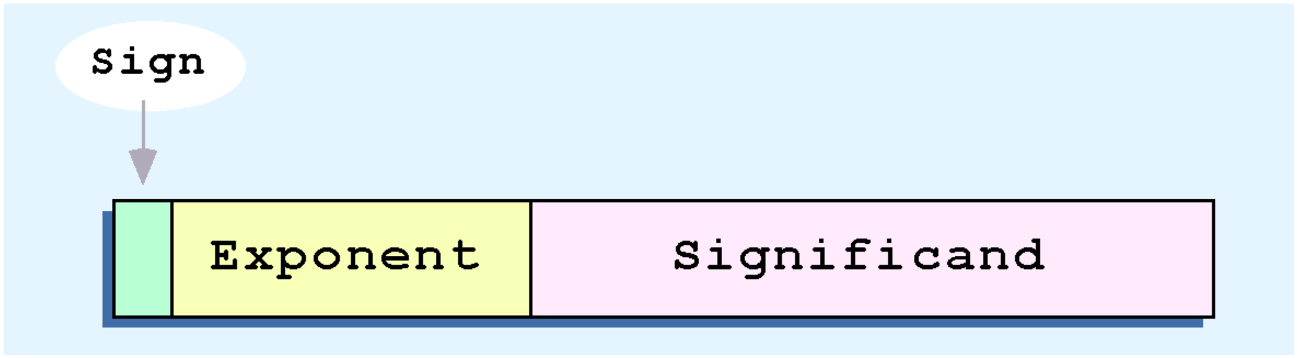
Note: Although "significand" and "mantissa" do not technically mean the same thing, many people use these terms interchangeably. We use the term "significand" to refer to the fractional part of a floating point number.

2.5 Floating-Point Representation



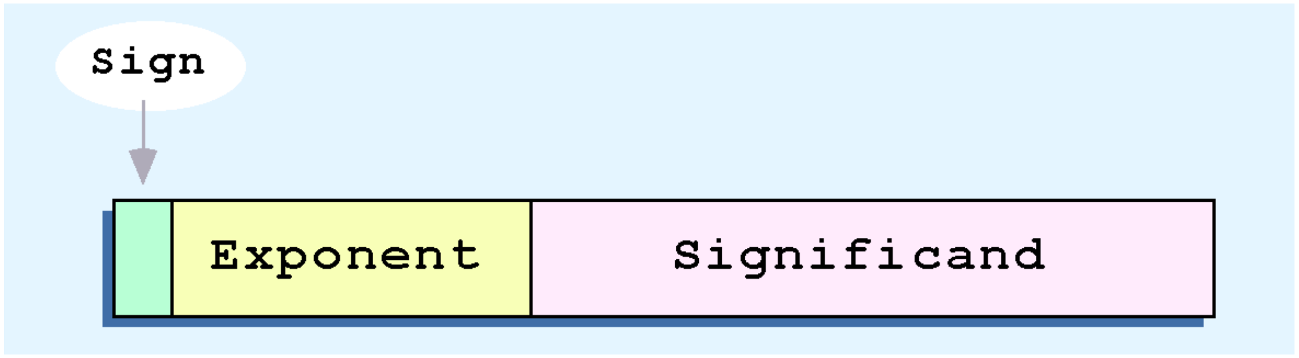
- The one-bit sign field is the sign of the stored value.
- The size of the exponent field determines the range of values that can be represented.
- The size of the significand determines the precision of the representation.

2.5 Floating-Point Representation



- We introduce a hypothetical "Simple Model" to explain the concepts
- In this model:
  - A floating-point number is 14 bits in length
  - The exponent field is 5 bits
  - The significand field is 8 bits

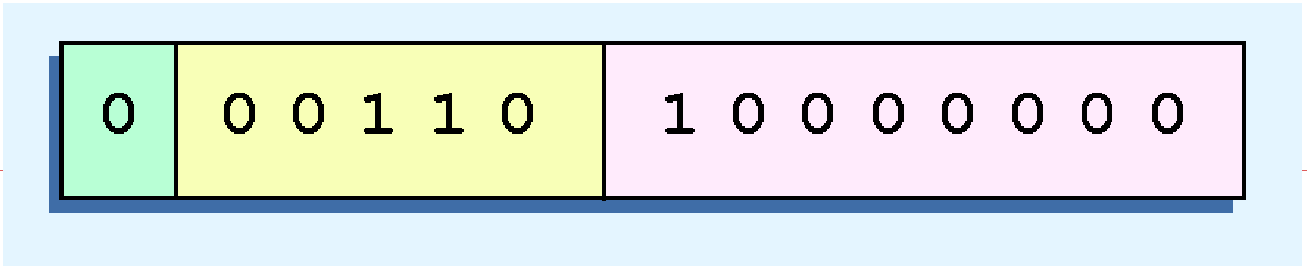
2.5 Floating-Point Representation



- The significand is always preceded by an implied binary point.
- Thus, the significand always contains a fractional binary value.
- The exponent indicates the power of 2 by which the significand is multiplied.

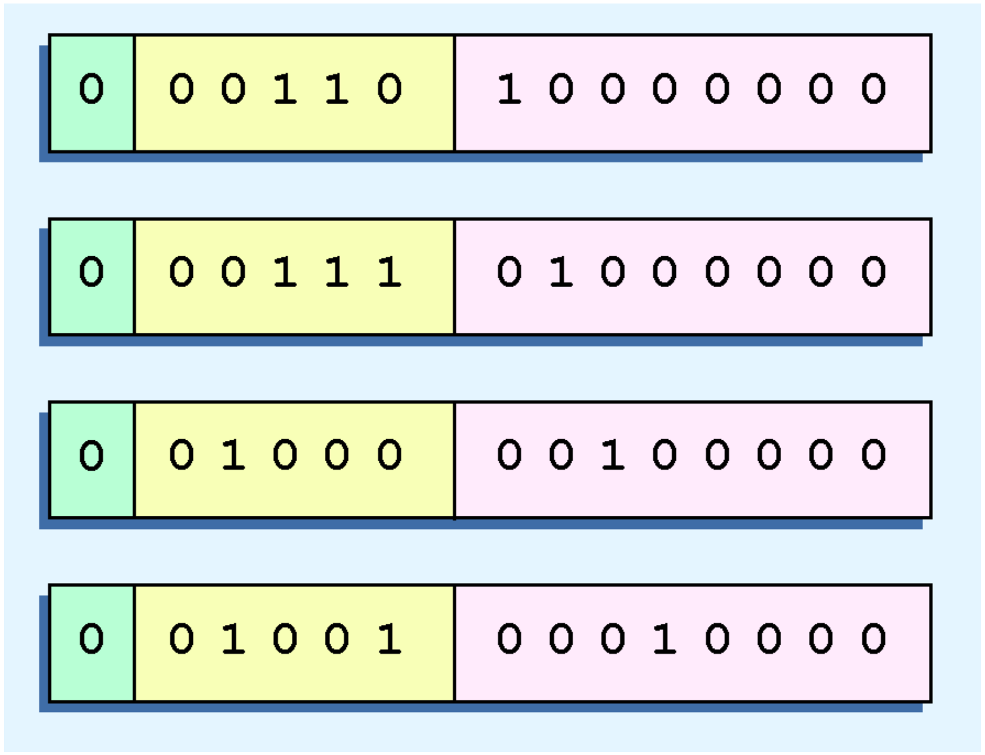
2.5 Floating-Point Representation

- Example:
  - Express 32_{10} in the simplified 14-bit floating-point model.
- We know that 32 is 2^5. So in (binary) scientific notation 32 = $1.0 \times 2^5 = 0.1 \times 2^6$.
  - In a moment, we'll explain why we prefer the second notation versus the first.
- Using this information, we put 110 (= 6_{10}) in the exponent field and 1 in the significand as shown.

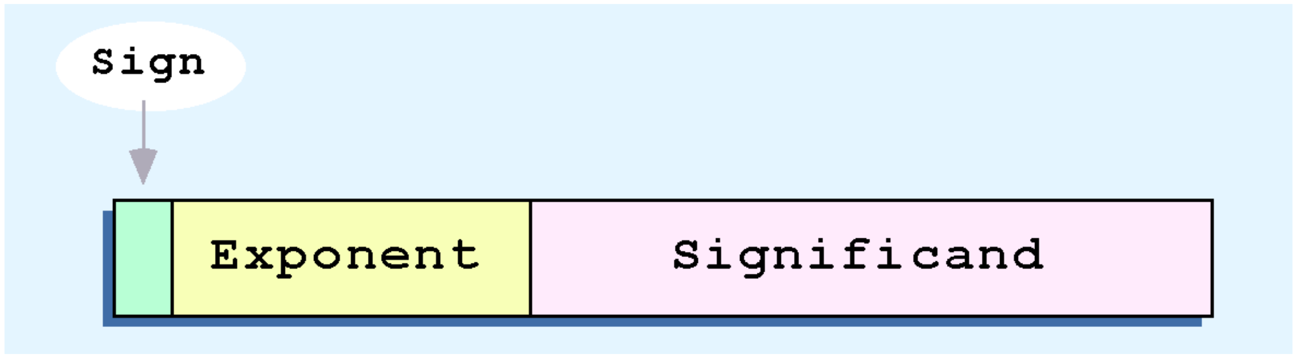


2.5 Floating-Point Representation

- The illustrations shown at the right are *all* equivalent representations for 32 using our simplified model.
- Not only do these synonymous representations waste space, but they can also cause confusion.



2.5 Floating-Point Representation



- Another problem with our system is that we have made no allowances for negative exponents. We have no way to express 0.5 (=2 $^{-1}$)! (Notice that there is no sign in the exponent field.)

All of these problems can be fixed with no changes to our basic model.

2.5 Floating-Point Representation

- To resolve the problem of synonymous forms, we establish a rule that the first digit of the significand must be 1, with no ones to the left of the radix point.
- This process, called *normalization*, results in a unique pattern for each floating-point number.
  - In our simple model, all significands must have the form 0.1xxxxxxxx
  - For example, 4.5 = 100.1 x 2^0 = 1.001 x 2^2 = 0.1001 x 2^3. The last expression is correctly normalized.

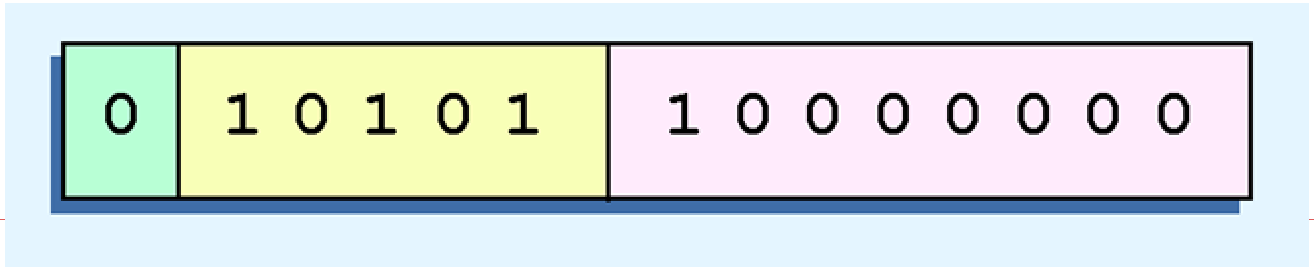
In our simple instructional model, we use no implied bits.

2.5 Floating-Point Representation

- To provide for negative exponents, we will use a *biased exponent*.
  - In our case, we have a 5-bit exponent.
  - $2^{5-1} - 1 = 2^4 - 1 = 15$
  - Thus will use 15 for our bias: our exponent will use *excess-15* representation.
- In our model, exponent values less than 15 are negative, representing fractional numbers.

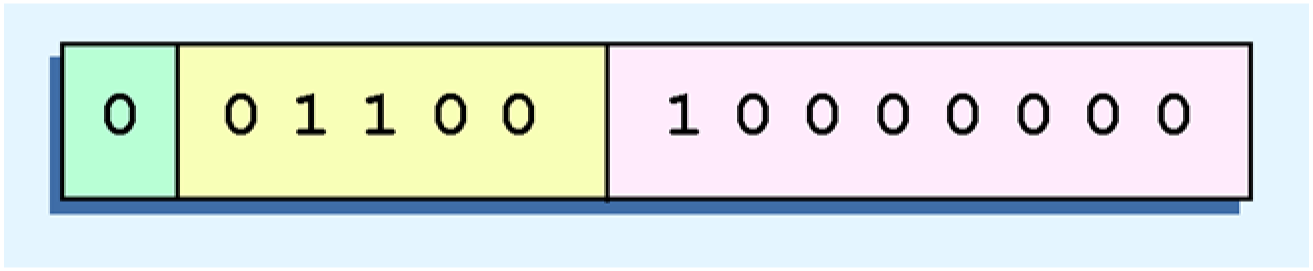
2.5 Floating-Point Representation

- Example:
  - Express 32_{10} in the revised 14-bit floating-point model.
- We know that $32 = 1.0 \times 2^5 = 0.1 \times 2^6$.
- To use our excess 15 biased exponent, we add 15 to 6, giving 21_{10} ($=10101_2$).
- So we have:



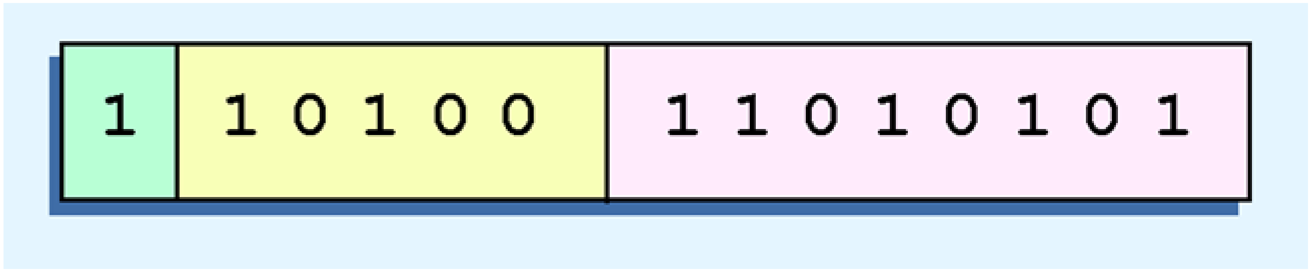
2.5 Floating-Point Representation

- Example:
  - Express 0.0625_{10} in the revised 14-bit floating-point model.
- We know that 0.0625 is 2^{-4}. So in (binary) scientific notation $0.0625 = 1.0 \times 2^{-4} = 0.1 \times 2^{-3}$.
- To use our excess 15 biased exponent, we add 15 to -3, giving 12_{10} ($=01100_2$).



2.5 Floating-Point Representation

- Example:
  - Express -26.625_{10} in the revised 14-bit floating-point model.
- We find $26.625_{10} = 11010.101_2$. Normalizing, we have: $26.625_{10} = 0.11010101 \times 2^5$.
- To use our excess 15 biased exponent, we add 15 to 5, giving 20_{10} ($=10100_2$). We also need a 1 in the sign bit.



2.5 Floating-Point Representation

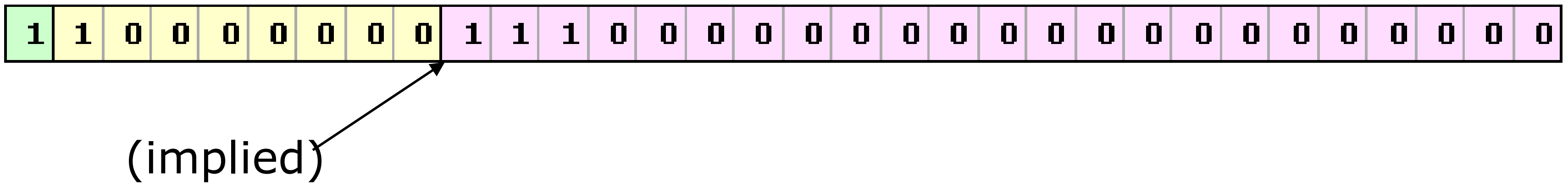
- The IEEE has established a standard for floating-point numbers
- The IEEE-754 *single precision* floating point standard uses an 8-bit exponent (with a bias of 127) and a 23-bit significand.
- The IEEE-754 *double precision* standard uses an 11-bit exponent (with a bias of 1023) and a 52-bit significand.

2.5 Floating-Point Representation

- In both the IEEE single-precision and double-precision floating-point standard, the significant has an implied 1 to the LEFT of the radix point.
  - The format for a significand using the IEEE format is: 1.xxx…
  - For example, 4.5 = .1001 x 2^3 in IEEE format is 4.5 = 1.001 x 2^2. The 1 is implied, which means is does not need to be listed in the significand (the significand would include only 001).

2.5 Floating-Point Representation

- Example: Express -3.75 as a floating point number using IEEE single precision.
- First, let's normalize according to IEEE rules:
  - 3.75 = -11.11_2 = -1.111×2^1
  - The bias is 127, so we add 127 + 1 = 128 (this is our exponent)
  - The first 1 in the significand is implied, so we have:



  - Since we have an implied 1 in the significand, this equates to $-(1).111_2 \times 2^{(128-127)} = -1.111_2 \times 2^1 = -11.11_2 = -3.75$.

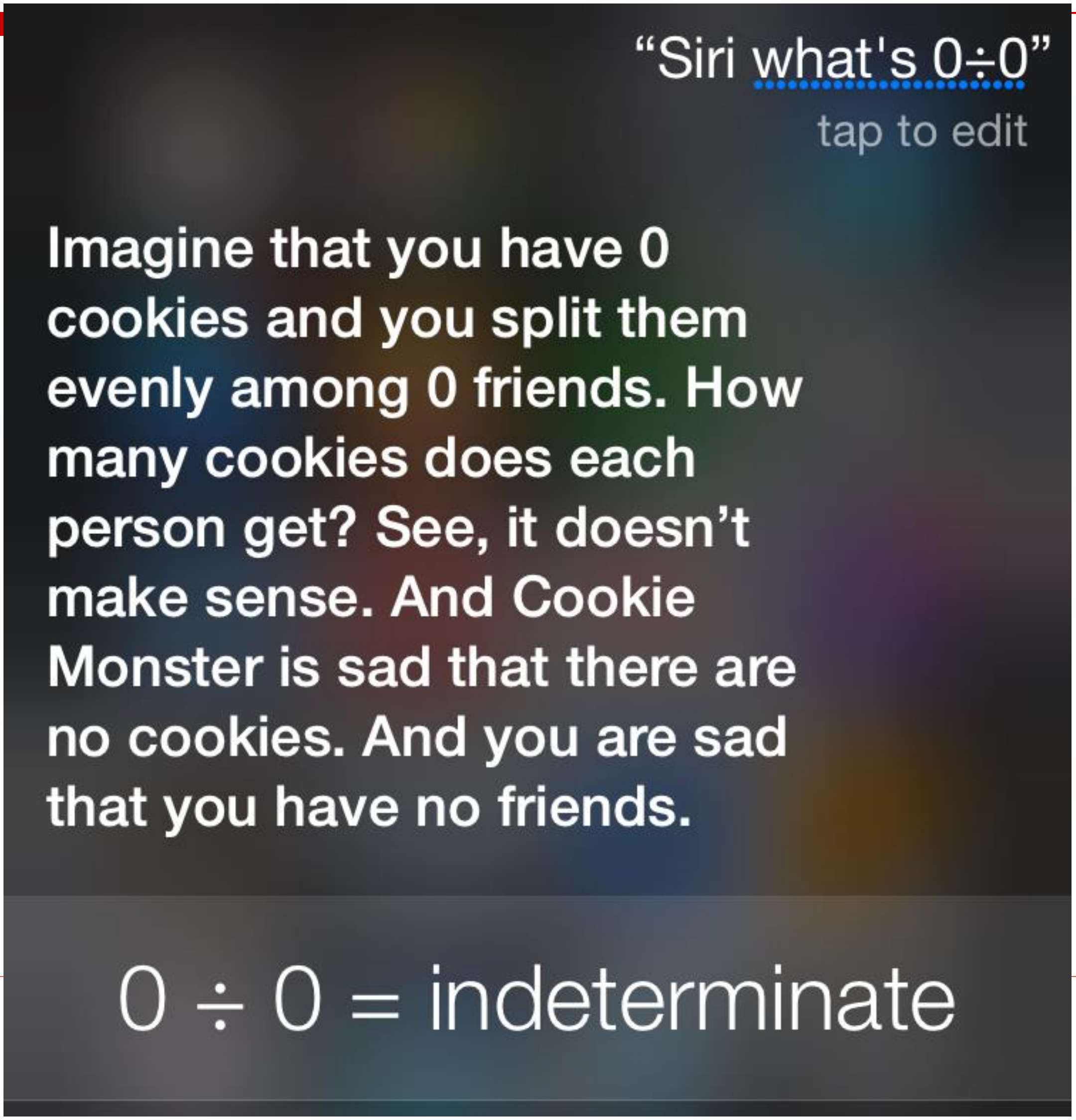
Exercise

- Use IEEE-754 single precision floating point standard to find binary representation of the following real number:
  - 0.0625
  - -26.625

2.5 Floating-Point Representation

- Using the IEEE-754 single precision floating point standard:
  - An exponent of 255 indicates a special value.
    - If the significand is zero, the value is $\pm$ infinity.
    - If the significand is nonzero, the value is NaN, "not a number," often used to flag an error condition.
- Using the double precision standard:
  - The "special" exponent value for a double precision number is 2047, instead of the 255 used by the single precision standard.

What is zero divided by zero?



"Siri what's 0÷0"

tap to edit

Imagine that you have 0 cookies and you split them evenly among 0 friends. How many cookies does each person get? See, it doesn't make sense. And Cookie Monster is sad that there are no cookies. And you are sad that you have no friends.

$0 \div 0 =$ indeterminate

2.5 Floating-Point Representation

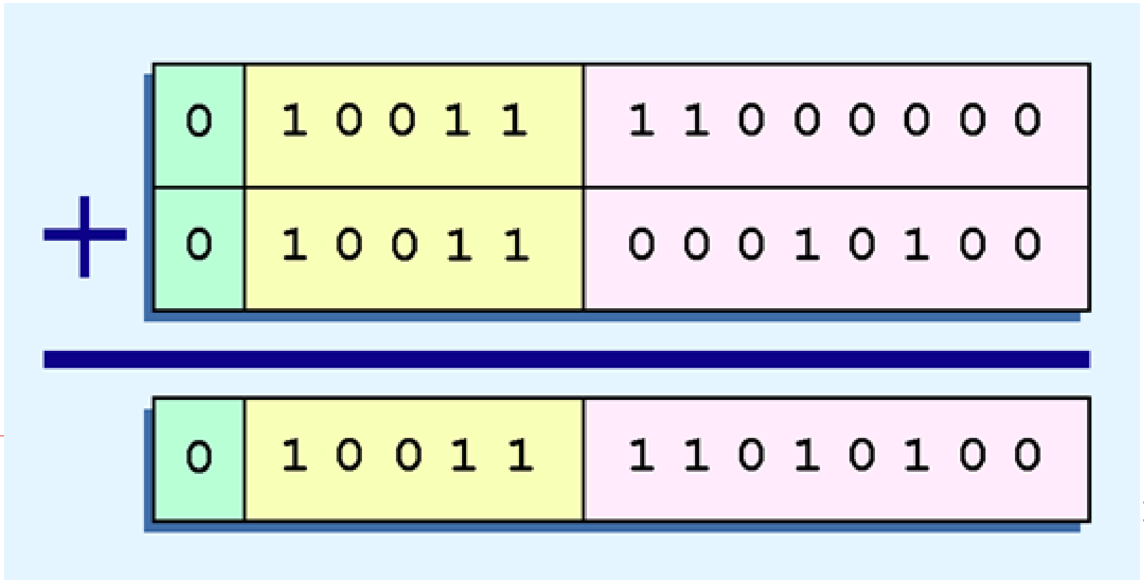
- Both the 14-bit model that we have presented and the IEEE-754 floating point standard allow two representations for zero.
  - Zero is indicated by all zeros in the exponent and the significand, but the sign bit can be either 0 or 1.
- This is why programmers should avoid testing a floating-point value for equality to zero.
  - Negative zero does not equal positive zero.

2.5 Floating-Point Representation

- Floating-point addition and subtraction are done using methods analogous to how we perform calculations using pencil and paper.
- The first thing that we do is express both operands in the same exponential power, then add the numbers, preserving the exponent in the sum.
- If the exponent requires adjustment, we do so at the end of the calculation.

2.5 Floating-Point Representation

- Example:
  - Find the sum of 12_{10} and 1.25_{10} using the 14-bit “simple” floating-point model.
- We find $12_{10} = 0.1100 \times 2^{4}$. And $1.25_{10} = 0.101 \times 2^{1} = 0.000101 \times 2^{4}$.
  - Thus, our sum is 0.110101×2^{4}.



2.5 Floating-Point Representation

- No matter how many bits we use in a floating-point representation, our model must be finite.
- The real number system is, of course, infinite, so our models can give nothing more than an approximation of a real value.
- At some point, every model breaks down, introducing errors into our calculations.
- By using a greater number of bits in our model, we can reduce these errors, but we can never totally eliminate them.

Examples

- Use JavaScript Console to compute the following:
  - 1.03 - 0.42
  - 1.00 - 9*0.1

Avoid float and double if exact answers are required!!!

Software Disaster

Software Disaster

Disasters Channel

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During the Gulf War in the early 1990's, Operation Desert Storm use...

https://www.youtube.com/watch?v=6OSfl7LMlJQ

2.5 Floating-Point Representation

- Our job becomes one of reducing error, or at least being aware of the possible magnitude of error in our calculations.
- We must also be aware that errors can compound through repetitive arithmetic operations.
- For example, our 14-bit model cannot exactly represent the decimal value 128.5. In binary, it is 9 bits wide:

$$10000000.1_2 = 128.5_{10}$$

2.5 Floating-Point Representation

- When we try to express 128.5_{10} in our 14-bit model, we lose the low-order bit, giving a relative error of:

$$\frac{128.5 - 128}{128.5} \approx 0.39\%$$

- If we had a procedure that repetitively added 0.5 to 128.5, we would have an error of nearly 2% after only four iterations.

2.5 Floating-Point Representation

- Floating-point errors can be reduced when we use operands that are similar in magnitude.
- If we were repetitively adding 0.5 to 128.5, it would have been better to iteratively add 0.5 to itself and then add 128.5 to this sum.
- In this example, the error was caused by loss of the low-order bit.
- Loss of the high-order bit is more problematic.

2.5 Floating-Point Representation

- Floating-point overflow and underflow can cause programs to crash.
- Overflow occurs when there is no room to store the high-order bits resulting from a calculation.
- Underflow occurs when a value is too small to store, possibly resulting in division by zero.

Experienced programmers know that it's better for a program to crash than to have it produce incorrect, but plausible, results.

2.5 Floating-Point Representation

- When discussing floating-point numbers, it is important to understand the terms *range*, *precision*, and *accuracy*.
- The range of a numeric integer format is the difference between the largest and smallest values that can be expressed.
- Accuracy refers to how closely a numeric representation approximates a true value.
- The precision of a number indicates how much information we have about a value

2.5 Floating-Point Representation

- Most of the time, greater precision leads to better accuracy, but this is not always true.
  - For example, 3.1333 is a value of pi that is accurate to two digits, but has 5 digits of precision.
- There are other problems with floating point numbers.
- Because of truncated bits, you cannot always assume that a particular floating point operation is associative or distributive.

2.5 Floating-Point Representation

- This means that we cannot assume:

  (a + b) + c = a + (b + c) or

  a*(b + c) = ab + ac

- Moreover, to test a floating point value for equality to some other number, it is best to declare a “nearness to x” epsilon value. For example, instead of checking to see if floating point x is equal to 2 as follows:

  if x = 2 then ...

  it is better to use:

  if (abs(x - 2) < epsilon) then ...

(assuming we have epsilon defined correctly!)

Outline

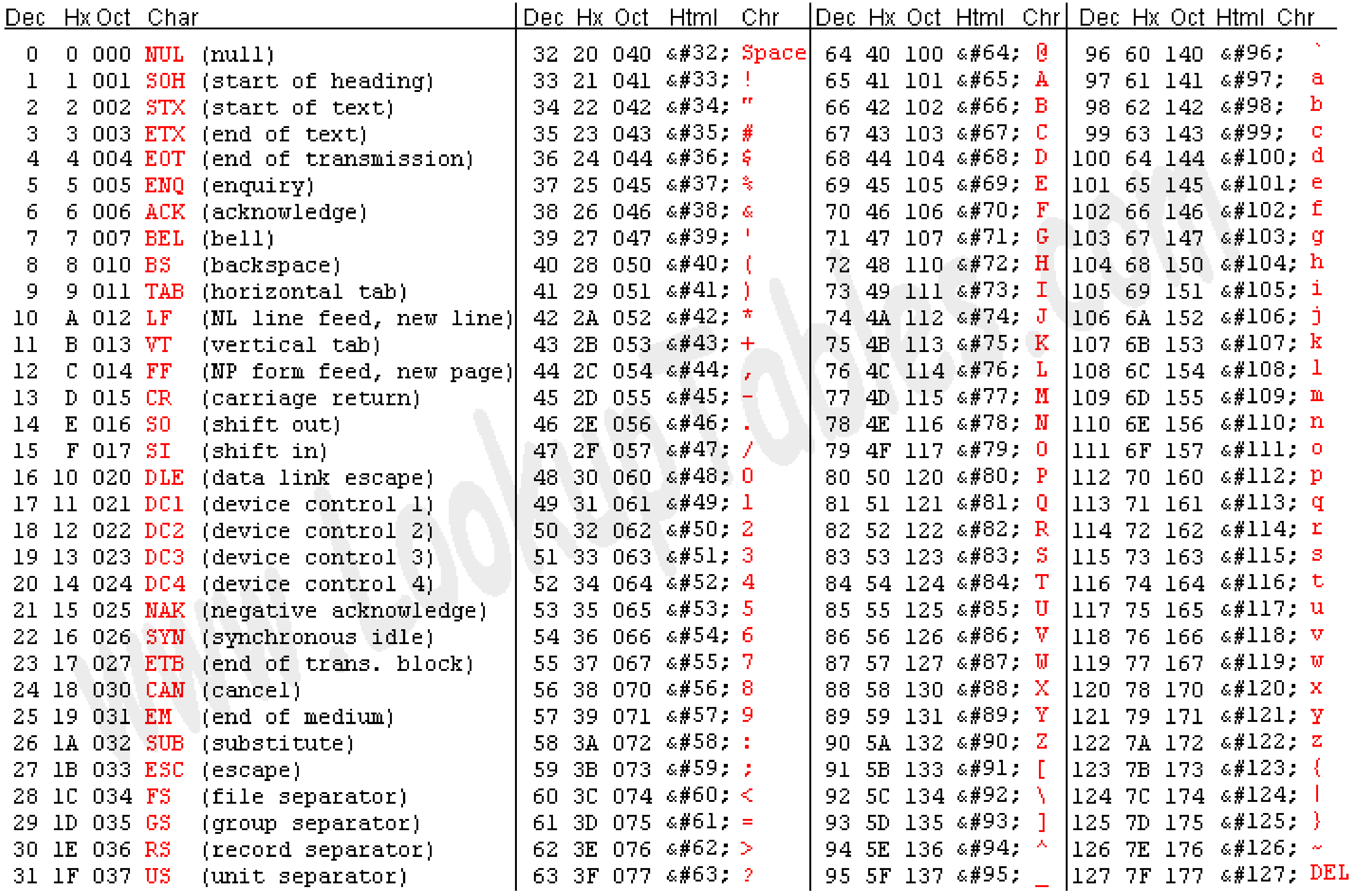
- Converting between different numeric-radix systems
- Binary addition and subtraction
- Two’s complement representation
- Floating-point representation
- **Characters in computer**

2.6 Characters in Computer

- You might say: "*Well, I know numbers can be represented as binary, what about characters?*"
- Characters are also represented as binary
  - However, all characters uses ASCII (/ˈæski/ ass-kee), as a character-encoding scheme.
  - ASCII --- American Standard Code for Information Interchange

ASCII Code

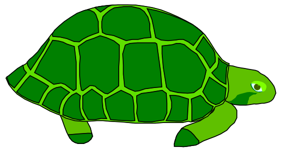
- It encodes 128 specified characters into 7-bit binary integers as shown by the ASCII chart.
  - The characters encoded are numbers 0 to 9, lowercase letters a to z, uppercase letters A to Z, basic punctuation symbols, control codes that originated with Teletype machines, and a space.
  - For the full ASCII table, see next page



Dec	Hx	Oct	Char		Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL	(null)	32	20	040	&#32;	Space	64	40	100	&#64;	@	96	60	140	&#96;	`
1	1	001	SOH	(start of heading)	33	21	041	&#33;	!	65	41	101	&#65;	A	97	61	141	&#97;	a
2	2	002	STX	(start of text)	34	22	042	&#34;	"	66	42	102	&#66;	B	98	62	142	&#98;	b
3	3	003	ETX	(end of text)	35	23	043	&#35;	#	67	43	103	&#67;	C	99	63	143	&#99;	c
4	4	004	EOT	(end of transmission)	36	24	044	&#36;	$	68	44	104	&#68;	D	100	64	144	&#100;	d
5	5	005	ENQ	(enquiry)	37	25	045	&#37;	%	69	45	105	&#69;	E	101	65	145	&#101;	e
6	6	006	ACK	(acknowledge)	38	26	046	&#38;	&	70	46	106	&#70;	F	102	66	146	&#102;	f
7	7	007	BEL	(bell)	39	27	047	&#39;	'	71	47	107	&#71;	G	103	67	147	&#103;	g
8	8	010	BS	(backspace)	40	28	050	&#40;	(	72	48	110	&#72;	H	104	68	150	&#104;	h
9	9	011	TAB	(horizontal tab)	41	29	051	&#41;	)	73	49	111	&#73;	I	105	69	151	&#105;	i
10	A	012	LF	(NL line feed, new line)	42	2A	052	&#42;	*	74	4A	112	&#74;	J	106	6A	152	&#106;	j
11	B	013	VT	(vertical tab)	43	2B	053	&#43;	+	75	4B	113	&#75;	K	107	6B	153	&#107;	k
12	C	014	FF	(NP form feed, new page)	44	2C	054	&#44;	,	76	4C	114	&#76;	L	108	6C	154	&#108;	l
13	D	015	CR	(carriage return)	45	2D	055	&#45;	-	77	4D	115	&#77;	M	109	6D	155	&#109;	m
14	E	016	SO	(shift out)	46	2E	056	&#46;	.	78	4E	116	&#78;	N	110	6E	156	&#110;	n
15	F	017	SI	(shift in)	47	2F	057	&#47;	/	79	4F	117	&#79;	O	111	6F	157	&#111;	o
16	10	020	DLE	(data link escape)	48	30	060	&#48;	0	80	50	120	&#80;	P	112	70	160	&#112;	p
17	11	021	DC1	(device control 1)	49	31	061	&#49;	1	81	51	121	&#81;	Q	113	71	161	&#113;	q
18	12	022	DC2	(device control 2)	50	32	062	&#50;	2	82	52	122	&#82;	R	114	72	162	&#114;	r
19	13	023	DC3	(device control 3)	51	33	063	&#51;	3	83	53	123	&#83;	S	115	73	163	&#115;	s
20	14	024	DC4	(device control 4)	52	34	064	&#52;	4	84	54	124	&#84;	T	116	74	164	&#116;	t
21	15	025	NAK	(negative acknowledge)	53	35	065	&#53;	5	85	55	125	&#85;	U	117	75	165	&#117;	u
22	16	026	SYN	(synchronous idle)	54	36	066	&#54;	6	86	56	126	&#86;	V	118	76	166	&#118;	v
23	17	027	ETB	(end of trans. block)	55	37	067	&#55;	7	87	57	127	&#87;	W	119	77	167	&#119;	w
24	18	030	CAN	(cancel)	56	38	070	&#56;	8	88	58	130	&#88;	X	120	78	170	&#120;	x
25	19	031	EM	(end of medium)	57	39	071	&#57;	9	89	59	131	&#89;	Y	121	79	171	&#121;	y
26	1A	032	SUB	(substitute)	58	3A	072	&#58;	:	90	5A	132	&#90;	Z	122	7A	172	&#122;	z
27	1B	033	ESC	(escape)	59	3B	073	&#59;	;	91	5B	133	&#91;	[	123	7B	173	&#123;	{
28	1C	034	FS	(file separator)	60	3C	074	&#60;	<	92	5C	134	&#92;	\	124	7C	174	&#124;	|
29	1D	035	GS	(group separator)	61	3D	075	&#61;	=	93	5D	135	&#93;	]	125	7D	175	&#125;	}
30	1E	036	RS	(record separator)	62	3E	076	&#62;	>	94	5E	136	&#94;	^	126	7E	176	&#126;	~
31	1F	037	US	(unit separator)	63	3F	077	&#63;	?	95	5F	137	&#95;	_	127	7F	177	&#127;	DEL

Source: www.LookupTables.com

- For example, lowercase j would become binary 1101010 (decimal 106) in ASCII.



54 68 65 20 45 6E 64

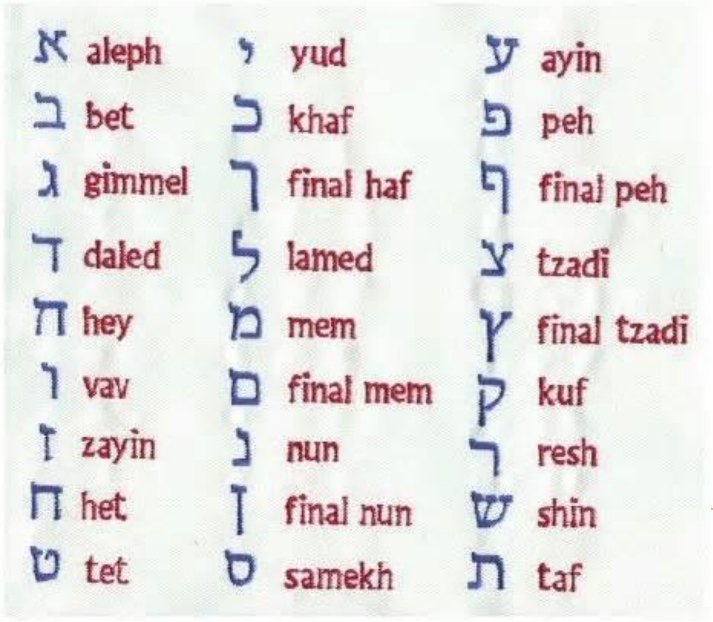
What do these hexadecimal numbers represent?

PPUNG TV
1
2
3
4
5
6
7

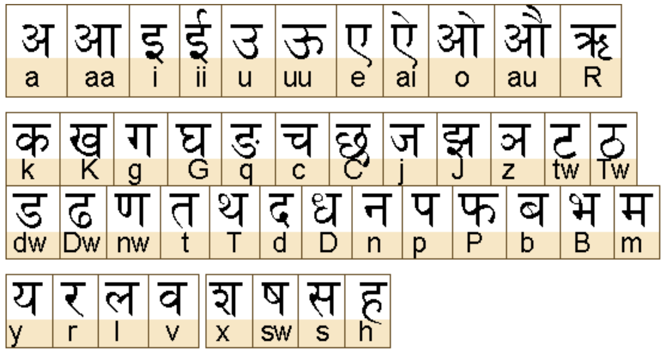
Is ASCII Enough?

- What about other characters, which is not English characters?
  - The Unicode will be needed

美 BEAUTY
清 CLARITY
英 COURAGE
命 DESTINY
花 FLOWER
恕 FORGIVENESS
朋友 FRIEND
恩 GRACE
福 HAPPINESS
和 HARMONY
貴 HONOR
愛 LOVE
月 MOON
平 PEACE
神 SPIRIT
星 STAR
日出 SUNRISE
安 TRANQUILITY
孚 TRUTH
智 WISDOM



א	aleph	י	yud	ע	ayin
ב	bet	כ	khaf	פ	peh
ג	gimmel	ך	final haf	ף	final peh
ד	daled	ל	lamed	צ	tzadi
ה	hey	מ	mem	ץ	final tzadi
ו	vav	ם	final mem	ק	kuf
ז	zayin	נ	nun	ר	resh
ח	het	ן	final nun	ש	shin
ט	tet	ס	samekh	ת	taf



अ	आ	इ	ई	उ	ऊ	ए	ऐ	ओ	औ	ऋ		
a	aa	i	ii	u	uu	e	ai	o	au	R		
क	ख	ग	घ	ङ	च	छ	ज	झ	ञ	ट	ठ	
k	K	g	G	q	c	C	j	J	z	tw	Tw	
ड	ढ	ण	त	थ	द	ध	न	प	फ	ब	भ	म
dw	Dw	nw	t	T	d	D	n	p	P	b	B	m
य	र	ल	व	श	ष	स	ह					
y	r	l	v	x	sw	s	h					

Unicode

- **Unicode** is a computing industry standard for the consistent encoding, representation, and handling of text expressed in most of the world's writing systems.
- The latest version of Unicode contains a repertoire of more than 110,000 characters covering 100 scripts and multiple symbol sets.
  - As of June 2014, the most recent version is *Unicode 7.0*. The standard is maintained by the Unicode Consortium.
- The most commonly used Unicode encodings are UTF-8, UTF-16 and the now-obsolete UCS-2.
  - UTF-8 uses one byte for any ASCII character, all of which have the same code values in both UTF-8 and ASCII encoding, and up to four bytes for other characters.
  - UTF-16 extends UCS-2, using one 16-bit unit for the characters that were representable in UCS-2 and two 16-bit units (4 × 8 bit) to handle each of the additional characters.
  - UCS-2 uses a 16-bit code unit (two 8-bit bytes) for each character, but cannot encode every character in the current Unicode standard.

http://www.w3schools.com/charsets/ref_utf_misc_symbols.asp

Chapter 2 Conclusion

- Computers store data in the form of bits, bytes, and words using the binary numbering system.
- Hexadecimal numbers are formed using four-bit groups called nibbles.
- Signed integers can be stored in one's complement, two's complement, or signed magnitude representation.
- Floating-point numbers are usually coded using the IEEE 754 floating-point standard.
- Floating-point operations are not necessarily commutative or distributive.
- Character data is stored using ASCII, EBCDIC, or Unicode.

End of Chapter 2