Given a weighted graph G, which is as follows:

22

36

3

1

10

10

16

3

3

18

1

10

4

1

16

4

21

16

4

13

3

8

9

6

17

4

27

4

130

1

1

16000000

8

8

9

a(i) a weighted adjacency list and

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| A | B, 10 | K, 3 |  |  |  |  |  |
| B | A, 10 | C, 36 |  |  |  |  |  |
| C | B, 36 | D, 3 |  |  |  |  |  |
| D | C, 3 | K, 16 | X, 1 |  |  |  |  |
| E | H, 10 | I, 3 | X, 22 |  |  |  |  |
| F | X, 10 |  |  |  |  |  |  |
| G | t, 9 |  |  |  |  |  |  |
| H | E, 10 | O, 4 | Z, 13 |  |  |  |  |
| h | O, 13 | P, 17 | R, 27 |  |  |  |  |
| I | E, 3 | N, 4 | n, 1 |  |  |  |  |
| J | K, 18 | L, 16 | n, 1 |  |  |  |  |
| K | A, 3 | D, 16 | J, 18 |  |  |  |  |
| L | J, 16 | Y, 3 |  |  |  |  |  |
| M | N, 21 | n, 4 |  |  |  |  |  |
| N | I, 4 | M, 21 | Z, 16 |  |  |  |  |
| n | I, 1 | J, 1 | M, 4 |  |  |  |  |
| O | H, 4 | h, 13 | T, 8 |  |  |  |  |
| P | h, 17 | S, 4 | Y, 6 |  |  |  |  |
| Q | Z, 4 |  |  |  |  |  |  |
| R | h, 13 |  |  |  |  |  |  |
| S | P, 4 | U, 9 | Y, 8 |  |  |  |  |
| T | O, 8 | U, 1 | V, 16 |  |  |  |  |
| t | G, 9 | U, 1 | W, 8 |  |  |  |  |
| U | S, 9 | T, 1 | t, 1 |  |  |  |  |
| V | T, 16 |  |  |  |  |  |  |
| W | t, 8 |  |  |  |  |  |  |
| X | D, 1 | E, 22 | F, 10 |  |  |  |  |
| Y | L, 3 | P, 6 | S, 8 |  |  |  |  |
| Z | H, 13 | N, 16 | Q, 4 |  |  |  |  |
|  |  |  |  |  |  |  |  |

a(i) Weighted adjacency list (29 vertices) of the graph.

a(ii) Weighted adjacency matrix (29 vertices) of the graph.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F | G | H | h | I | J | K | L | M | N | n | O | P | Q | R | S | T | t | U | V | W | X | Y | Z |
| A |  | 10 |  |  |  |  |  |  |  |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| B | 10 |  | 36 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| C |  | 36 |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| D |  |  | 3 |  |  |  |  |  |  |  |  | 16 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |
| E |  |  |  |  |  |  |  | 10 |  | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 22 |  |  |
| F |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 10 |  |  |
| G |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 9 |  |  |  |  |  |
| H |  |  |  |  | 10 |  |  |  |  |  |  |  |  |  |  |  | 4 |  |  |  |  |  |  |  |  |  |  |  | 13 |
| h |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 | 17 |  | 27 |  |  |  |  |  |  |  |  |  |
| I |  |  |  |  | 3 |  |  |  |  |  |  |  |  |  | 4 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| J |  |  |  |  |  |  |  |  |  |  |  | 18 | 16 |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| K | 3 |  |  | 16 |  |  |  |  |  |  | 18 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| L |  |  |  |  |  |  |  |  |  |  | 16 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 3 |  |
| M |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 21 | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| N |  |  |  |  |  |  |  |  |  | 4 |  |  |  | 21 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 16 |
| n |  |  |  |  |  |  |  |  |  | 1 | 1 |  |  | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| O |  |  |  |  |  |  |  | 4 |  | 13 |  |  |  |  |  |  |  |  |  |  |  | 8 |  |  |  |  |  |  |  |
| P |  |  |  |  |  |  |  |  | 17 |  |  |  |  |  |  |  |  |  |  |  | 4 |  |  |  |  |  |  | 6 |  |
| Q |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 4 |
| R |  |  |  |  |  |  |  |  | 27 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| S |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 4 |  |  |  |  |  | 9 |  |  |  | 8 |  |
| T |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 8 |  |  |  |  |  |  | 1 | 16 |  |  |  |  |
| t |  |  |  |  |  |  | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  | 8 |  |  |  |
| U |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 9 | 1 | 1 |  |  |  |  |  |  |
| V |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 16 |  |  |  |  |  |  |  |
| W |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 8 |  |  |  |  |  |  |
| X |  |  |  | 1 | 22 | 10 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Y |  |  |  |  |  |  |  |  |  |  |  |  | 3 |  |  |  |  | 6 |  |  | 8 |  |  |  |  |  |  |  |  |
| Z |  |  |  |  |  |  |  | 13 |  |  |  |  |  |  | 16 |  |  |  | 4 |  |  |  |  |  |  |  |  |  |  |
|  | A | B | C | D | E | F | G | H | h | I | J | K | L | M | N | n | O | P | Q | R | S | T | t | U | V | W | X | Y | Z |

**The DFS traversal’s stack with time-stamp**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Nodes(pushOrder, PopOffOrder) | NodeReached, Weights | NodeReached, Weights | NodeReached, Weights |  |  |  |  |
| A ( 1, 29) | B, 10 | K, 3 |  |  |  |  |  |
| B (29, 25) | A, 10 be | C, 36 |  |  |  |  |  |
| C (28, 26) | B, 36 | D, 3 |  |  |  |  |  |
| D ( 3, 27) | C, 3 | K, 16 | X, 1 |  |  |  |  |
| E ( 6, 23) | H, 10 | I, 3 | X, 22 |  |  |  |  |
| F ( 5, 1) | X, 10 |  |  |  |  |  |  |
| G (17, 3) | t, 9 |  |  |  |  |  |  |
| H (20, 9) | E, 10 be | O, 4 | Z, 13 |  |  |  |  |
| h (25, 11) | O, 13 | P, 17 be | R, 27 |  |  |  |  |
| I ( 7, 22) | E, 3 | N, 4 | n, 1 |  |  |  |  |
| J ( 9, 20) | K, 18 be | L, 16 | n, 1 |  |  |  |  |
| K ( 2, 28) | A, 3 | D, 16 | J, 18 |  |  |  |  |
| L (10, 19) | J, 16 | Y, 3 |  |  |  |  |  |
| M (24, 6) | N, 21 | n, 4 be |  |  |  |  |  |
| N (23, 7) | I, 4 be | M, 21 | Z, 16 |  |  |  |  |
| n ( 8, 21) | I, 1 | J, 1 | M, 4 |  |  |  |  |
| O (19, 12) | H, 4 | h, 13 | T, 8 |  |  |  |  |
| P (12 17) | h, 17 | S, 4 | Y, 6 |  |  |  |  |
| Q (22, 5) | Z, 4 |  |  |  |  |  |  |
| R (26, 10) | h, 13 |  |  |  |  |  |  |
| S (13, 16) | P, 4 | U, 9 | Y, 8 be |  |  |  |  |
| T (18, 14) | O, 8 | U, 1 | V, 16 |  |  |  |  |
| t (15, 4) | G, 9 | U, 1 | W, 8 |  |  |  |  |
| U (14, 15) | S, 9 | T, 1 | t, 1 |  |  |  |  |
| V (27, 13) | T, 16 |  |  |  |  |  |  |
| W (16, 2) | t, 8 |  |  |  |  |  |  |
| X ( 4, 24) | D, 1 | E, 22 | F, 10 |  |  |  |  |
| Y (11, 18) | L, 3 | P, 6 | S, 8 |  |  |  |  |
| Z (21, 8) | H, 13 | N, 16 | Q, 4 |  |  |  |  |
|  |  |  |  |  |  |  |  |

**“be” signified a back edge is first found.**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| PUSHDOWN\_STACK | | | | | | | | | | | |
| Node(PushOrdered, PopOffOrdered | | | | | | | | | | | |
|  |  |  |  |  |  |  | |  |  | | |
|  |  |  |  |  |  |  | |  |  | | |
|  |  |  |  |  |  |  | |  |  | | |
|  |  |  |  | M(24, 6) |  |  | |  |  | | |
|  |  |  | Q(22, 5) | N(23, 7) |  |  | |  |  | | |
|  |  |  | Z(21, 8) | Z((21, 8) | R(26, 10) |  | |  |  | | |
|  |  |  | H(20, 9) | H(20, 9) | h(25, 11) |  | |  |  | | |
|  | W(16, 2) | G(17, 3) | O(19, 12) | O(19, 12) | O(19, 12) | V(27, 13) | |  |  | | |
|  | t (15, 4) | t (15, 4) | T(18, 14) | T(18, 14) | T(18, 14) | T(18, 14) | |  |  | | |
|  | U(14, 15) | U(14, 15) | U(14, 15) | U(14, 15) | U(14, 15) | U(14, 15) | |  |  | | |
|  | S(13, 16) | S(13, 16) | S(13, 16) | S(13, 16) | S(13, 16) | S(13, 16) | |  |  | | |
|  | P(12 17) | P(12 17) | P(12 17) | P(12 17) | P(12 17) | P(12 17) | |  |  | | |
|  | Y(11, 18) | Y(11, 18) | Y(11, 18) | Y(11, 18) | Y(11, 18) | Y(11, 18) | |  |  | | |
|  | L(10, 19) | L(10, 19) | L(10, 19) | L(10, 19) | L(10, 19) | L(10, 19) | |  |  | | |
|  | J( 9, 20) | J( 9, 20) | J( 9, 20) | J( 9, 20) | J( 9, 20) | J( 9, 20) | |  |  | | |
|  | n( 8, 21) | n( 8, 21) | n( 8, 21) | n( 8, 21) | n( 8, 21) | n( 8, 21) | |  |  | | |
|  | I( 7, 22) | I( 7, 22) | I( 7, 22) | I( 7, 22) | I( 7, 22) | I( 7, 22) | |  |  | | |
| F( 5, 1) | E( 6, 23) | E( 6, 23) | E( 6, 23) | E( 6, 23) | E( 6, 23) | E( 6, 23) | | B(29, 25) |  | | |
| X( 4, 24) | X( 4, 24) | X( 4, 24) | X( 4, 24) | X( 4, 24) | X( 4, 24) | X( 4, 24) | | C(28, 26) |  | | |
| D( 3, 27) | D( 3, 27) | D( 3, 27) | D( 3, 27) | D( 3, 27) | D( 3, 27) | D( 3, 27) | | D( 3, 27) |  | | |
| K( 2, 28) | K( 2, 28) | K( 2, 28) | K( 2, 28) | K( 2, 28) | K( 2, 28) | K( 2, 28) | | K( 2, 28) |  | | |
| A( 1,29) | A( 1,29) | A( 1,29) | A( 1,29) | A( 1,29) | A( 1,29) | A( 1,29) | | A( 1,29) |  | | |
| NIL | NIL | NIL | NIL | NIL | NIL | NIL | NIL | | | NIL |

The two orderings of vertices yielded by the DFS are as follows:

**Push Onto Order:**

A( 1,29), K( 2, 28), D( 3, 27), X( 4, 24), F( 5, 1), E( 6, 23), I( 7, 22), n( 8, 21)¸ J( 9, 20),

L(10, 19), Y(11, 18), P(12 17), S(13, 16), U(14, 15), t (15, 4), W(16, 2), G(17, 3), T(18, 14),

O(19, 12), H(20, 9), Z((21, 8), Q(22, 5), N(23, 7), M (24, 6), h(25, 11), R(26, 10), V(27, 13),

C(28, 26), B(29, 25)

**Pop Off Order**

F( 5, 1), W(16, 2), G(17, 3), t (15, 4), Q(22, 5), M(24, 6), N(23, 7), Z((21, 8), H(20, 9),

R(26, 10), h(25, 11), O(19, 12), V(27, 13), T(18, 14), U(14, 15), S(13, 16), P(12 17), Y(11, 18),

L(10, 19), J( 9, 20), n( 8, 21)¸ I( 7, 22), E( 6, 23), X( 4, 24), B(29, 25), C(28, 26), D( 3, 27),

K( 2, 28), A( 1,29),

(5,1)

(1.29)

(29,25)

(3,27) ))

(28,26)

(6.23)

22

36

3

1

10

10

3

3

16

(7,22) ))

(2,28) ))

(8,21) ))

18

1

10

4

(9,20) ))

(23,7)))

1

16

4

21

16

(10,19) ))

(24,6)))

4

(11,18) ))

13

3

(21,8) ))

(13,16) ))

(22,5)))

8

(25,11) ))

(20,9))

9

6

(14,15) ))

17

4

27

4

(12,17) ))

(18,14))

1

1

130

(26,10) ))

(15,4))

16000000

8

8

9

(19,12))

(17,3))

(16,2))

(27,13))

**c DFS’s tree**

(2,28) ))

3

(1.29)

16

(4,24) ))

(3,27) ))

1

10

(5,1)

3

221111

**W=30**

3

36

**W=58**

1

1

16

**W=148**

3

21

6

**W=115**

16

4

4

9

13

27

13

8

1

1

**W=134**

4

16

9

8

**W=102**

**W=95**

**W=94**

(5,1)

(29,25)

(1, 29)

10

(3,27) ))

(28,26)

(6.23)

22

36

3

1

10

(4,24) ))

3

3

16

(7,22) ))

(2,28) ))

(8,21) ))

18

1

10

4

(9,20) ))

(23,7)))

1

16

4

21

16

(10,19) ))

(24,6)))

4

(11,18) ))

13

3

(21,8) ))

(13,16) ))

(22,5)))

8

(25,11) ))

(20,9))

9

6

(14,15) ))

17

4

27

4

(12,17) ))

(18,14))

1

1

130

(26,10) ))

(15,4))

16000000

8

8

9

(19,12))

(17,3))

(16,2))

(27,13))

**d The articulation points are D, X, U, T, Z, h, t.**

(2,28) ))

3

(1.29)

16

(4,24) ))

(3,27) ))

1

10

(5,1)

3

221111

**W=30**

3

36

**W=58**

1

1

16

**W=148**

3

21

6

**W=115**

16

4

4

9

13

27

13

8

1

1

**W=134**

4

16

9

8

**W=102**

**W=95**

**W=94**

Traversing the graph given in Problem 7, based on its **weighted adjacency list** representation obtained in a(i), construct its **breath-first search (BFS) tree** forest **starting from vertex A**. For this, you need to use a **queue** (note the *difference from DFS*) to trace the operation of breadth-first search, indicating the order in which the vertices {…, V’, V”, … } were visited. i.e., the order of the operation of **adding several vertices to**, or **removing a vertex from the queue** {,…, , , …}. The order in which vertices are added to the queue (i.e., enqueue operation) is the same order in which they are removed from it (i.e., dequeue operation). Indicate the **tree edges** (indicated as solid lines) and **cross-edges** (indicated as dotted lines) for your trees. For this problem, you need to answer a through e, which is as follows:

a. Show the traversal’s queue with a time stamp indicating the order in which the vertices were visited. What is the ordering of vertices yielded by the BFS?

b. Construct the corresponding breadth-first search (BFS) tree forest, with an indication of tree edges and cross edges in addition to back edges and forward edges)

c. From the obtained BFS tree forest, compute the *shortest* *distance (smallest number of edges) from* A to vertex G.

d. What are the time efficiency and space efficiency of the BFS?

1. **The BFS traversal’s queue with a time-stamp**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Nodes(pushOrder, PopOffOrder) | NodeReached, Weights | NodeReached, Weights | NodeReached, Weights |  |  |  |  |
| A 1 | B, 10 | K, 3 |  |  |  |  |  |
| B 3 | A, 10 | C, 36 |  |  |  |  |  |
| C 6 | B, 36 | D, 3 |  |  |  |  |  |
| D 4 | C, 3 c | K, 16 | X, 1 |  |  |  |  |
| E 11 | H, 10 | I, 3 c | X, 22 |  |  |  |  |
| F 10 | X, 10 |  |  |  |  |  |  |
| G 29 | t, 9 |  |  |  |  |  |  |
| H 15 | E, 10 | O, 4 | Z, 13 |  |  |  |  |
| h 21 | O, 13 | P, 17 | R, 27 |  |  |  |  |
| I 12 | E, 3 | N, 4 | n, 1 |  |  |  |  |
| J 5 | K, 18 | L, 16 | n, 1 |  |  |  |  |
| K 2 | A, 3 | D, 16 | J, 18 |  |  |  |  |
| L 9 | J, 16 | Y, 3 |  |  |  |  |  |
| M 13 | N, 21 f | n, 4 c |  |  |  |  |  |
| N 16 | I, 4 | M, 21 | Z, 16 c |  |  |  |  |
| n 8 | I, 1 | J, 1 | M, 4 |  |  |  |  |
| O 19 | H, 4 | h, 13 c | T, 8 |  |  |  |  |
| P 17 | h, 17 | S, 4 c | Y, 6 |  |  |  |  |
| Q 24 | Z, 4 |  |  |  |  |  |  |
| R 25 | h, 13 |  |  |  |  |  |  |
| S 18 | P, 4 | U, 9 | Y, 8 |  |  |  |  |
| T 23 | O, 8 | U, 1 | V, 16 |  |  |  |  |
| t 26 | G, 9 | U, 1 | W, 8 |  |  |  |  |
| U 22 | S, 9 | T, 1 f | t, 1 |  |  |  |  |
| V 27 | T, 16 |  |  |  |  |  |  |
| W 28 | t, 8 |  |  |  |  |  |  |
| X 7 | D, 1 | E, 22 | F, 10 |  |  |  |  |
| Y 14 | L, 3 | P, 6 | S, 8 |  |  |  |  |
| Z 20 | H, 13 | N, 16 | Q, 4 |  |  |  |  |
|  |  |  |  |  |  |  |  |

**“c” signified a cross edge is first found. “f” is a forward edge.**

**FIFO Queue Ordering**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| A | K | B | D | J | C | X | n | L | F | E | I | M | Y | H |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |  |
| N | P | S | O | Z | h | U | T | Q | R | t | V | W | G |  |

b. Construct the corresponding breadth-first search (BFS) tree forest, with an indication of tree edges and cross edges in addition to back edges and forward edges)

10

6

3

1

4

11

7

22

36

3

1

10

10

12

3

3

16

2

8

18

1

10

4

5

1

16

16

4

21

13

16

20

9

4

24

13

3

14

18

8

21

15

9

6

17

4

27

4

22

23

17

1

1

130

26

25

16000000

8

8

9

19

29

27

28

Dotted lines are crossing edges (c).

5

1

2

3

10

3

18

36

c

16

4

5

6

16

46

1

1

8

7

9

3

22

4

1

10

11

12

13

14

c

10

10

8

f

26

30

4

6

17

16

15

c

18

1

f

27

17

9

4

13

19

22

130

21

20

c

4

1

27

8

24

25

26

90

8

23

f

69

9

16000000

29

28

27

80

66

67

BFS Tree. Cross edges are shown in dotted lines. Arrow is used to indicate when a node (the tail of the arrow) is visited, it can reach the other node (the head of the arrow) which is already visited.

c. From the obtained BFS tree forest, compute the *shortest* *distance (smallest number of edges) from* A to vertex G.

All the paths with weights generated by BFS are as follows:

A K D X F (30 )

A K D X E H O T V (80)

A K D X E H Z Q (69)

A D J n I N (27)

A D J n M (26)

A K J L Y P h R (90)

A K J L Y S U t W (66)

A K J L Y S U t G (67) The shortest distance (smallest number of edges) from A to G.

A B C (46)

c. The shortest path from A to G is 67

A K J L Y S U t G

1 18 16 3 8 9 1 9

This compares against 59, the min spanning tree constructed by Prim’s Algorithm

d. What are the time efficiency and space efficiency of the BFS?

Check these

**The time and space Efficiency of the DFS for the adjacent lists is Θ(|V| + |E| )**

**The time and space Efficiency of the DFS for the adjacent Matrix is Θ(|V| 2 )**

From the given graph G, given a source vertex A, use Prim’s algorithm to find the minimum spanning tree for the graph. For each step, state the set of tree vertices **VT** that are visited, and the set of remaining vertices **V - VT**, that have not visited. The A(-, -) is to denote it is a starting node A (or called a source node A). K(A, 3) is to denote the node K can be reached by node A, which is already visited, where the distance from A to K is 3. More importantly, you need to give a table stating the **tree vertices** and **remaining vertices** with their weights (i.e., the corresponding edges with their weights). You do not have to give the intermediate graph, but you have to show the final minimum spanning tree (via highlight edges) within the graph given in problem 7.

a. Compute the minimum spanning tree of the given graph G.

|  |  |  |
| --- | --- | --- |
| Tree Vertices | Remaining Vertices | The set of **VT** and the set of **V - VT** |
| A(-, -) | B(A, 10), K(A, 3), ?(-, ∞ ) |  |
| K(A, 3) | . . . |  |
|  |  |  |

where VT = { A } and

V - VT = { B, K, ?}, where “?” is to denote any vertex in the graph, which is not adjacent to A.

If you wish, use the symbol “?” to denote all the vertices in the graph which are not adjacent to every vertex in VT.

b. For the given graph G, what is your obtained minimum spanning tree? What is the total weight of the obtained minimum spanning tree? [Highlight the obtained (from a) minimum spanning tree in the given graph G.]

Given the source node A, list all the paths of the obtained minimum spanning tree from source node A to their leaf nodes. And then compute the total weight of each branch of the paths. For example, the branch A, K, D, X, F with a weight of 30.

c. From your obtained minimum spanning tree, what is the minimum distance from vertex A to vertex G?

Given a weighted graph G, which is as follows:

22

36

3

1

10

10

3

3

16

18

1

10

4

1

16

4

21

16

4

13

3

8

9

6

17

4

27

4

1

1

130

16000000

8

8

9

A weighted adjacency list for the given graph G is as follows

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| A(-, -) | B, 10 | K, 3 |  |  |  |  |  |
| B(A, 10) | A, 10 | C, 36 |  |  |  |  |  |
| C(D, 3) | B, 36 | D, 3 |  |  |  |  |  |
| D(K, 16) | C, 3 | K, 16 | X, 1 |  |  |  |  |
| E(I, 3) | H, 10 | I, 3 | X, 22 |  |  |  |  |
| F(X, 10) | X, 10 |  |  |  |  |  |  |
| G(t, 9) | t, 9 |  |  |  |  |  |  |
| H(E, 10) | E, 10 | O, 4 | Z, 13 |  |  |  |  |
| h(O, 13) | O, 13 | P, 17 | R, 27 |  |  |  |  |
| I(n, 1) | E, 3 | N, 4 | n, 1 |  |  |  |  |
| J(K, 18) | K, 18 | L, 16 | n, 1 |  |  |  |  |
| K(A, 3) | A, 3 | D, 16 | J, 18 |  |  |  |  |
| L(Y, 3) | J, 16 | Y, 3 |  |  |  |  |  |
| M(n, 4) | N, 21 | n, 4 |  |  |  |  |  |
| N(I, 4) | I, 4 | M, 21 | Z, 16 |  |  |  |  |
| n(J, 1) | I, 1 | J, 1 | M, 4 |  |  |  |  |
| O(H, 4) | H, 4 | h, 13 | T, 8 |  |  |  |  |
| P(S, 4) | h, 17 | S, 4 | Y, 6 |  |  |  |  |
| Q(Z, 4) | Z, 4 |  |  |  |  |  |  |
| R(h, 13) | h, 13 |  |  |  |  |  |  |
| S(U, 9) | P, 4 | U, 9 | Y, 8 |  |  |  |  |
| T(O, 8) | O, 8 | U, 1 | V, 16 |  |  |  |  |
| t(U, 1) | G, 9 | U, 1 | W, 8 |  |  |  |  |
| U(T, 1) | S, 9 | T, 1 | t, 1 |  |  |  |  |
| V(T, 16) | T, 16 |  |  |  |  |  |  |
| W(t, 8) | t, 8 |  |  |  |  |  |  |
| X(D, 1) | D, 1 | E, 22 | F, 10 |  |  |  |  |
| Y(P, 6) | L, 3 | P, 6 | S, 8 |  |  |  |  |
| Z(H, 13) | H, 13 | N, 16 | Q, 4 |  |  |  |  |
|  |  |  |  |  |  |  |  |

a.(i) Weighted adjacency list (29 vertices) of the graph.

a\_soln: Compute the minimum spanning tree of the given graph G.

|  |  |  |
| --- | --- | --- |
| Tree Vertices | Remaining Vertices | Let G contains all the 29 given nodes |
| A(-, -) | K(A, 3), B(A, 10), ?(-, ∞ ) | VT = { A } and  V - VT = { G - A}, |
| K(A, 3) | B(A, 10), D(K,16), J(K,18), ?(-, ∞ ) | VT = { A, K } and  V - VT = { G – {A, K}}, |
| B(A, 10) | D(K,16), J(K,18), C(B, 36), ?(-, ∞ ) | VT = { A, K, B } and  V - VT = { G – {A, K, B}}, |
| D(K,16) | X(D, 1), C(D, 3), J(K,18), C(B, 36), ?(-, ∞ ) | VT = { A, K, B, D } and  V - VT = { G – {A, K, B, D}}, |
| X(D, 1) | C(D, 3), F(X, 10), J(K,18), E(X, 22), C(B, 36), ?(-, ∞ ) | VT = { A, K, B, D, X } and  V - VT = { G – {A, K, B, D, X}}, |
| C(D, 3) | F(X, 10), J(K,18), E(X, 22), ~~C(B, 36),~~ ?(-, ∞ )  Strike out C(B, 36) since B and C are in VT. | VT = { A, K, B, D, X, C} and  V - VT = { G – {A, K, B, D, X, C}}, |
| F(X, 10) | J(K,18), E(X, 22), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F} and  V - VT = { G – {A, K, B, D, X, C, F}}, |
| J(K,18) | n(J, 1), L(J, 16), E(X, 22), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J} and  V - VT = { G – {A, K, B, D, X, C, F, J}}, |
| n(J, 1) | I(n, 1), M(n, 4), L(J, 16), E(X, 22), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n} and  V - VT = { G – {A, K, B, D, X, C, F, J, n}}, |
| I(n, 1) | E((I, 3), M(n, 4), N(I, 4), L(J, 16), E(X, 22), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I}}, |
| E(I, 3) | M(n, 4), N(I, 4), H(E, 10), L(J, 16), ~~E(X, 22),~~ ?(-, ∞ )  Strike out E(X, 22) since both X and E nodes are in VT | VT = { A, K, B, D, X, C, F, J, n, I, E} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E}}, |
| M(n, 4) | N(I, 4), H(E, 10), L(J, 16), N(M, 21), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M}}, |
| N(I, 4), | H(E, 10), L(J, 16), Z(N, 16), ~~N(M, 21),~~ ?(-, ∞ )  Strike out N(M, 21), since both M and N are in VT. | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N}}, |
| H(E, 10) | O(H, 4), Z(H, 13), L(J, 16), Z(N, 16), ?(-, ∞ )  Do not create X(E, 22), since both X and E are in VT. | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H}}, |
| O(H, 4), | T(O, 8), Z(H, 13), h(O, 13), L(J, 16), Z(N, 16), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O}}. |
| T(O, 8) | U(T, 1), Z(H, 13), h(O, 13), L(J, 16), Z(N, 16), V(T, 16), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T}}. |
| U(T, 1) | t(U, 1), S(U, 9), Z(H, 13), h(O, 13), L(J, 16), Z(N, 16), V(T, 16), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U}}. |
| t(U, 1) | W(t, 8), S(U, 9), G(t, 9), Z(H, 13), h(O, 13), L(J, 16), Z(N, 16), V(T, 16), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t}}. |
| W(t, 8) | S(U, 9), G(t, 9), Z(H, 13), h(O, 13), L(J, 16), Z(N, 16), V(T, 16), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S}}. |
| S(U, 9) | P(S, 4), Y(S, 8), G(t, 9), Z(H, 13), h(O, 13), L(J, 16), Z(N, 16), V(T, 16), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S}}. |
| P(S, 4) | Y(P, 6), Y(S, 8), G(t, 9), Z(H, 13), h(O, 13), L(J, 16), Z(N, 16), V(T, 16), h(P, 17), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P}}. |
| Y(P, 6) | L(Y, 3), ~~Y(S, 8),~~ G(t, 9), Z(H, 13), h(O, 13), L(J, 16), Z(N, 16), V(T, 16), h(P, 17), ?(-, ∞ )  Strike out Y(S, 8) since both S and Y are in VT. | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P, Y} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P, Y}}. |
| L(Y, 3) | G(t, 9), Z(H, 13), h(O, 13), ~~L(J, 16),~~ Z(N, 16), V(T, 16), h(P, 17), ?(-, ∞ )  Do not create J(L, 16) since both L and J are in VT. | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P, Y, L} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P, Y, L}}. |
| G(t, 9), | Z(H, 13), h(O, 13), Z(N, 16), V(T, 16), h(P, 17), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P, Y, L, G} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P, Y, L, G}}. |
| Z(H, 13), | Q(Z, 4), h(O, 13), ~~Z(N, 16),~~ V(T, 16), h(P, 17), ?(-, ∞ )  Strike out Z(N, 16), since both N and Z are in VT. | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P, Y, L, G, Z} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P, Y, L, G, Z}}. |
| Q(Z, 4) | h(O, 13), V(T, 16), h(P, 17), ?(-, ∞ ) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P, Y, L, G, Z, Q} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P, Y, L, G, Z, Q}}. |
| h(O, 13), | V(T, 16), ~~h(P, 17),~~ R(h, 27), ?(-, ∞ )  Strike out h(P, 17) since both h and P are in VT. | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P, Y, L, G, Z, Q, h} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P, Y, L, G, Z, Q, h}}. |
| V(T, 16), | R(h, 27) | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P, Y, L, G, Z, Q, h, V} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P, Y, L, G, Z, Q, h, V}}. |
| R(h, 27) |  | VT = { A, K, B, D, X, C, F, J, n, I, E, M, N, H, O, T, U, t, W, S, P, Y, L, G, Z, Q, h, V, R} and  V - VT = { G – {A, K, B, D, X, C, F, J, n, I, E. M, N, H, O, T, U, t, W, S, P, Y, L, G, Z, Q, h, V, R}} = . |
|  |  |  |

Let V or G contains all 29 nodes.

A(-, -)

VT = { A } and

V - VT = { G - A}.

K(A, 3)

VT = { A, K } and

V - VT = { G – {A, K}}.

B(A, 10)

VT = { A, K, B } and

V - VT = { G – {A, K, B}}.

D(K,16)

VT = { A, K, B, D } and

V - VT = { G – {A, K, B, D}}.

…

b: For the given graph G, what is your obtained minimum spanning tree? What is the total weight of the obtained minimum spanning tree? [Highlight the obtained (from a) minimum spanning tree in the given graph G.]

Given the source node A, list all the paths of the obtained minimum spanning tree from source node A to their leaf nodes. And then compute the total weight of each branch of the paths. For example, the branch A, K, D, X, F with a weight of 30.

Applying Prim’s Algorithm on the given weighted graph G, the minimum spanning tree is generated and is as follows:

36 c

22 c

3

1

10

10

3

3

16

18

1

10

4

1

21 c

16 f,b

4

16 c

4

13

3

8 f,b

17 c

9

6

4

27

4

1

1

130

16000000

8

8

9

The obtained minimum spanning tree has a total weight which is 210

The obtained minimum spanning tree is as follows:

10

10

3

18

16

36 c

1

3

22

4

1

1

26

3

4

10

21 c

22 c

30

27

10

16 c

13

4

16 f,b

8

13

4

16

53

1

64

27

1

80

17 c

9

4

9

8

58

59

8 f,b

6

71

3

All the paths of the branches from the source-node A with their (weights) are as follows:

**A** 10 **B** (10).

**A** 3  **K** 16 **D** 3 **C** (22)

**A** 3 **K**  16 **D** 1 **X** 10 **F** (30)

**A** 3 **K** 18 **J** 1 **n**  4 **M** (26)

**A** 3 **K** 18 **J**  1 **n** 1 I 4 **N** (27)

**A** 3 **K** 18 **J** 1 **n** 1 **I** 3 **E** 10 **H**  13 **Z** 4 **Q** (53)

**A** 3 **K** 18 **J** 1 **n**  1 **I** 3 **E** 10 **H**  4 **O** 13 **h** 27 **R**  (80)

**A** 3 **K** 18 **J** 1 **n** 1  **I** 3 **E**  10 **H**  4 **O** 8 **T** 16 **V** (64)

**A**  3 **K** 18 **J** 1 **n** 1  **I** 3 **E**  10 **H** 4 **O** 8 **T** 1 **U**  1 **t**  8 **W** (58)

**A** 3 **K** 18 **J** 1 **n**  1 **I** 3 **E** 10 **H** 4 **O** 8 **T** 1 **U** 1 **t** 8 **G**  (59)

**A** 3 **K** 18 **J** 1 **n** 1  **I** 3 **E** 10 **H** 4 **O** 8 **T** 1 **U** 9 **S** 4 **P**  6 **Y** 3  **L** (71)

c: From your obtained minimum spanning tree, what is the minimum distance from vertex A to vertex G?

The shortest distance from A to G is the path

**A**  3 **K**  18 **J** 1 **n**  1 **I**  3 **G** 10 **H** 4 **O**  8 **T** 1 **U**  1 **t**  9 **G**  (59),

which has a total weight of 59.

**12/21/2022**