WS 4 Intensity and Irradiance.

Imagine that you have a point source. If you place a small sheet of paper of area $\mathbf{A}$ such that it is perpendicular to the light (at least in center) a distance $r$ from the source, how could you calculate the irradiance on the surface?


Now imagine that we rotate this sheet of paper so that its normal makes an angle with respect to the central ray of light. What would happen to the number of rays that intersect the surface? What would happen to the irradiance? Explain.

Given your answers to this question, consider the sun-earth system. What can you say about the amount of optical power/unit area delivered to the earth as you go from the equator to the poles? Why would the axial tilt of the earth be related to the seasons?

Solid angle: To understand solid angle, you must understand where radians come from. Consider a length of $\operatorname{arc} \mathbf{s}$. How is the length of $s$ related to $r$ and $\theta$ ?


What can you say about the arc length and r (how they are oriented with respect to each other?

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If we write out a differential form for this we get: $r d \theta=d s$. To find the total arc length we would integrate around an angle so that we have $s=\int r d \theta$ which going around full circle we get $s=r \theta=r(2 \pi)$. So a full circle is $2 \pi$ radians. Ultimately how do you relate the angle $\theta$ to arc length and radius?

Now consider a slightly different situation. The arc length $\mathbf{s}$ is not perpendicular everywhere to the radii. Therefore can we really say that $s=r \theta$ ? The dashed line indicates the original arc.


In this situation we can see that $r$ is really a function of $\theta$. Therefore if we were to calculate the arc length we would have to integrate: $s=\int r(\theta) d \theta$. However, what if we wanted to figure out the angle $\theta$ ? Is there a relationship between the dashed curve and s if we imagine that the angle is small?

For solid angle, the definition is similar to that or an angle except that we are concerned with an area rather than an arc length. We want to relate the solid angle ( $\Omega$ ) to a surface area (A) and a radius of curvature. Come up with an expression relating these three quantities being certain to be dimensionally correct. The units of solid angle are steradian.

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The basic relationship between solid angle, radius and area is given by: $A=r^{2} \Omega$, as long as the area is perpendicular to the radius. We might say $d A_{\perp}=r^{2} d \Omega$. We need to be concerned with the area that is perpendicular to the radius. The Area of a surface is given by $A=\int r^{2} d \Omega$.

If we imagine a sphere, the surface area of a sphere is given by $4 \pi r^{2}=r^{2} \Omega$ which means that in going around all angles the solid angle is given by $4 \pi$ steradians.

Intensity is the Power per unit steradian. How does this differ from the irradiance?

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Now imagine that we rotate this sheet of paper so that its normal makes an angle with respect to the central ray of light. What would happen to the number of rays that intersect the surface? What would happen to the intensity? Explain.

