What is polarization?

Consider an EM wave. Suppose we write the electric field portion of the wave as $\vec{E}_1(x,t) = E_o \;\; \cos\theta\hat{x} + \sin\theta\hat{y} \;\; e^{i(kz-\omega t)}$. In what direction is this wave traveling? Explain.

Write an expression for the electric field at a single point. How would you describe this electric field?

Find a coordinate system in which this wave is parallel to the new x-axis and write the wave in that coordinate system.

Because this electric field can be written as __________ to a ____________ axis for ______ times,

this light is known as __________ polarized light.

Suppose that there was a second EM wave in the same coordinate system as in the preceding section. The electric field has the form $\vec{E}_2(x,t) = \frac{1}{2} E_o \;\; \cos\phi\hat{x} + \sin\phi\hat{y} \;\; e^{i(kz-\omega t)}$. This wave overlaps the initial one. What is the impact of this wave on the electric field at a single point? Explain.

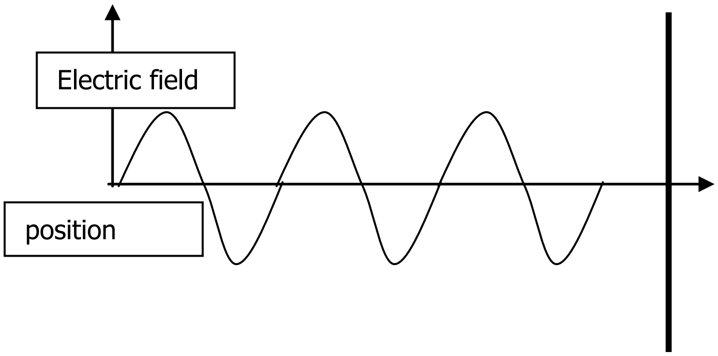
What can you say about the wavelength, frequency and relative phase of the two waves?

Write an expression for the electric field at that single point as a function of time.

At a single point in time and space, the electric field is given by the ___________ of __________ the

_____________ ____________ _____________ at that point.

If we define a particular coordinate system, we can define horizontal and vertical polarizations of light. It is also important to pay attention to the relative phases of the two waves.



Consider the drawing above. We have the electric field of an EM wave traveling to the right. There is an antenna which is aligned with the electric field. What will happen to the wire as the wave travels past the wire?

What impact will this have on the wave?

Now imagine that we turn the antenna so that it is perpendicular to the electric field. What impact will the wave have on the wire now? What impact on the wave?

Suppose that the antenna were oriented at an angle θ with respect to the electric field. What impact will this have on the wave?

Now imagine that you had a grid of very closely spaced, fine wires. How would this effect a wave passing through a grid arranged at 45 degrees from the orientation of the electric field? Be specific.

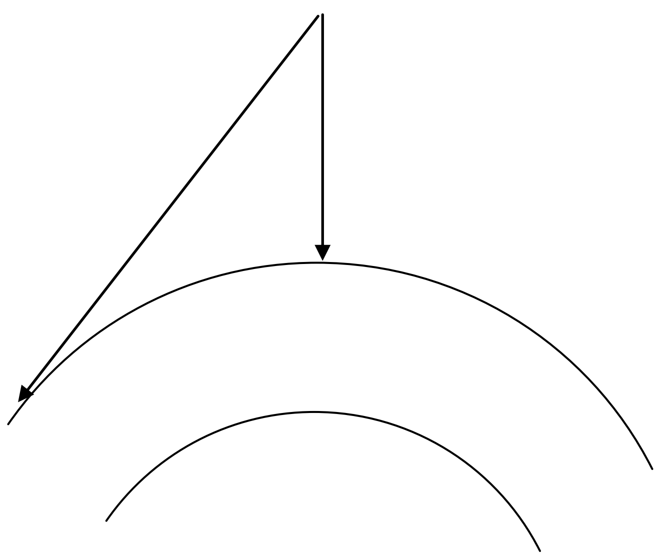
This is an example of **dichroism**. Dichroism takes place in nature in that the crystaline structure is such that one particular direction of electric field is strongly absorbed while one perpendicular to that is not (principle axis). You might think of the structure as if an optically active electron is bound in three dimensions by 6 springs. The spring constants are not the same so that for some particular angle it is possible to set the electron into resonance, while not in resonance for the angle perpendicular to this.

A polarizing sheet is made by stretching long chains of organic molecules and adding iodine. These chains of molecules act as wires similar to a wire grid polarizer on a molecular level.

We have also seen polarization by reflection. What is this type reflection known as? Approximately how much light is reflected under these conditions? What could you do to improve the amount of reflection.

Polarization of oscillating charges.
Consider a vibrating charge, the basic method of producing an EM wave. If the charge is vibrating in the plane of the paper (oriented from top of the page to the bottom of the page) what are the directions of radiation? How do you know?

If the charge were to vibrate in and out of the plane of the paper, what would be the directions of emission?



Now consider scattered light. Scattered light is produced by an incident wave causing the electrons bound to atoms to oscillate. These oscillating charges emit in particular directions. Based on this information, why is the blue sky polarized – more so the farther from the sun towards the horizon?

Linear polarizers
A linear polarizer could be thought of as a device that defines a polarization axis. It does this because of its atomic/molecular structure which makes it interact more strongly with a particular electric field orientation.

Law of Malus

Imagine that you have one linear polarizer followed by a second at some angle. We will call the second polarizer the analyzer because it is rotated with respect to the first.

If the angle between the first and second polarizer is θ, what fraction of the light that passes through the first polarizer is able to pass through the second? Explain your answer.

Matrix notation

We can write the sum of two linear polarizations in a given coordinate system as $\vec{E}_T(x,t) = \; E_{ox}\hat{x} + E_{oy}\hat{y} \;\; e^{i(kz-\omega t)}$ We could also represent this as a matrix multiplication:

$$\vec{E}_T(x,t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} E_{ox}\hat{x} \\ E_{oy}\hat{y} \end{bmatrix} e^{i(kz-\omega t)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1\hat{x} \\ \frac{E_{oy}}{E_{ox}}\hat{y} \end{bmatrix} E_{ox} e^{i(kz-\omega t)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{\sqrt{a^2+b^2}} \begin{bmatrix} a\hat{x} \\ b\hat{y} \end{bmatrix} E_{T0} e^{i(kz-\omega t)}$$

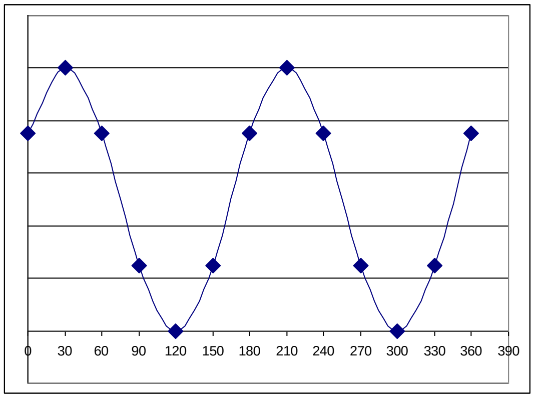
What polarizations are represented by: $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$?

Starting with one of those two polarizations listed above, suppose we replace the 2x2 matrix by one that is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. What impact does this have on an incident wave with both polarizations? What does this matrix represent?

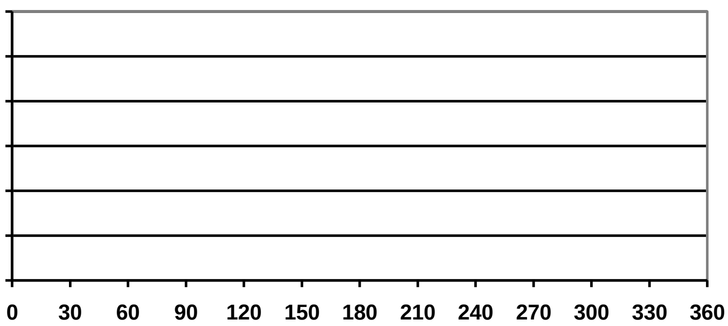
Starting with vertically polarized light (y direction), suppose you have a matrix such as $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ What optical element does this matrix represent?

What is unpolarized light?

Consider that you have a linear polarizer and at 0 degrees the polarizer is oriented vertically. Given an unknown light source, you rotate the polarizer and observe the transmission of the light. What can you say about the polarization of this light? Explain.



What must be true about unpolarized when viewed through a linear polarizer so that it be considered unpolarized light. Sketch what you might observe for unpolarized light using a rotating polarizer

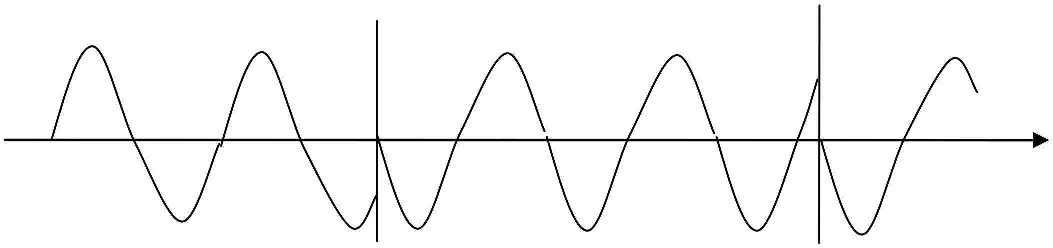


What do you believe is unpolarized light?

Saying light is unpolarized is really a misnomer. The reason it is a misnomer is that it is _________ for

the ______________ _______________ to not have some ____________________ at any instant in time.

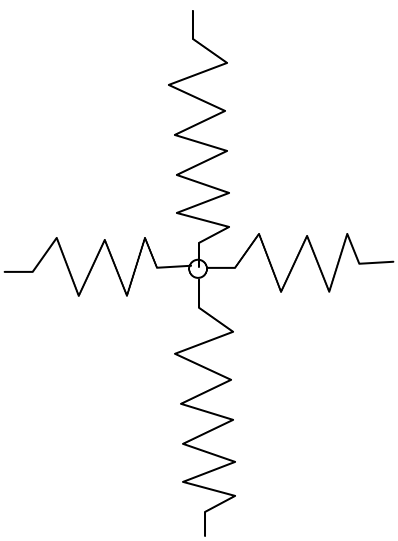
Consider a light source which produces nominally unpolarized light. There are many atoms and molecules involved in the production of the emission. The atoms that are close together are likely to be radiating in phase. However as we move from that point, the correlation between atoms diminishes and they can be oscillating in very different directions. Additionally, we have to consider how long an atom can radiate before having that radiation be interrupted by interaction with other atoms. The wave that is produced between these interactions is known as a **wave train**. In each of the interactions the orientation of the vibrating charge could be changed, changing the polarization of the emission. In an incoherent source, the wave trains tend to be very short.



Furthermore, if you consider scattering from larger particles such as in clouds or rough surfaces, the light is depolarized. Why would the depolarization happen?

Try setting up a laser and placing a polarizer in front of the laser, then rotate the polarizer. Is the beam polarized? Now allow the laser beam to impinge upon a piece of paper (or ground glass or plastic) and look at the reflected/scattered light through the polarizer. Is this light polarized?

Consider a material in which there is a structure so that in one particular direction, the optically active electron is more strongly bound in one direction than another. The figure below schematically indicates this bound electron. Mechanically, the springs are of the same material, therefore the shorter



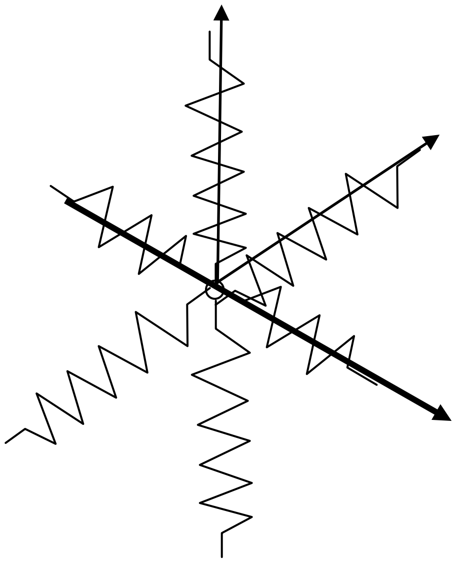
springs have a _______________ spring constant. This means that the resonant frequency in the

horizontal direction is _______________ than in the vertical direction. This in turn implies that the index of refraction for a frequency lower

than either resonance will be ________________ in the horizontal direction than in the vertical direction.

This is known as **birefringence.** Birefringence corresponds to a material that has different indices of refraction and different speeds of the wave depending upon the polarization of the incident light. It could be said that there is an anisotropy in the binding force on the electron.

In reality the optically active charges would be connected in 3 dimensions (see diagram below). If you had a crystal that had symmetry in two directions and an asymmetry in the third (with regards to binding force), then the asymmetric direction would be called the **optic axis** (heavy line in diagram). Suppose you had light incident in the direction of the optic axis with polarizations that would lie along each of the two planes of vibration. What would the impact of the crystal be on light with both vertical and horizontal polarizations?



Imagine that a beam of polarized light was incident upon one of these materials but perpendicular to the optic axis. In this case, one of the polarizations would lie along the stronger interaction axis and one along the weaker interaction axis.

Because the light interacts more strongly with one particular polarization, what will happen to the light with each of the two polarizations?

We will, for now, assume that axis with the weaker interaction (which means that it has its resonance at lower frequencies and therefore will interact more with the light causing a higher index of refraction for that polarization) This will be the slow axis and we will have that be in the vertical direction (the material is defining the polarization). The fast axis will be in the direction in which there is a stronger interaction and we will say that is on the horizontal axis.

By varying the thickness of the material, we can advance or retard the phase of some component of the incident wave. Suppose that light with equal horizontal and vertical that we were able to change the effective path length of one component of the incident wave by a full wavelength. What would happen to the polarization of the exiting light? Write an expression representing the wave before and after the crystal.

Now suppose that the phase of the wave is changed by ½ of a wavelength relative to the other. What would happen to the polarization of the exiting light? Write an expression representing the wave before and after the crystal.

Suppose that there is a retardation of ¼ of wavelength for one of the two components. What would happen to the polarization of the exiting light? Write an expression for electric field as a function of time.

This situation corresponds to circularly polarized light. This is because the electric field is rotating as a function of time. If you were to rotate a linear polarizer, you would not see any variation in the intensity with angle of the polarizer.

We can represent a circular polarizer in a matrix by:
$e^{-i\frac{\pi}{4}}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$, $e^{i\frac{\pi}{4}}\begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}$. If you have light linearly polarized at 45 degrees incident on each of these calculate the exiting EM wave. Which component is retarded in each case?

What do each of these matrices do?
$e^{-i\frac{\pi}{2}}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$, $e^{i\frac{\pi}{2}}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

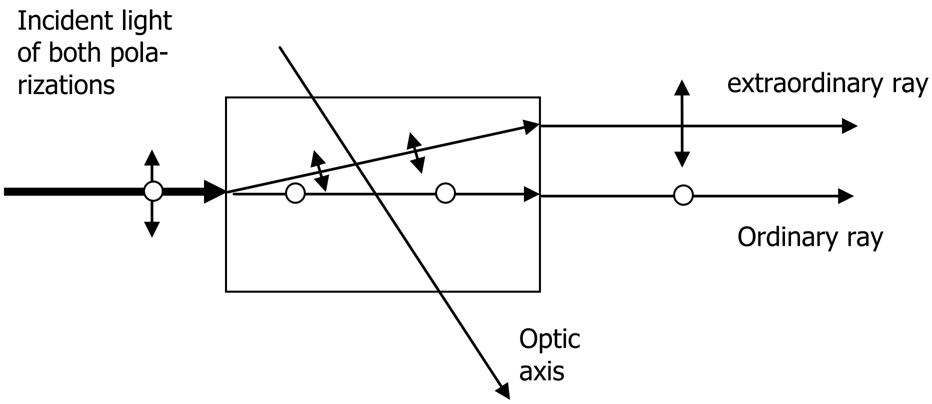
A general form for the retardation of the different components is

$$\begin{bmatrix}e^{i\varepsilon_x} & 0\\ 0 & e^{i\varepsilon_y}\end{bmatrix}$$

Suppose that we retard the phase by some amount greater or smaller than π/4? What impact does that have on the polarization? How would you identify light that has this polarization?

Suppose that the light is incident on a ¼ wave plate (circular polarizer), but not at an angle of 45 degrees. What is the polarization of the exiting light?

Imagine that light were incident upon the crystal but at an angle to the optic axis rather than perpendicular or parallel. If we think of the index of refraction as being part of a complex scattering process, then the oscillating charge will produce two separate waves, one traveling faster than the other. This is drastically different from the scattering process in homogeneous mediums. What will be observed will be a second beam that will travel in a different direction from the incident beam. One component will be perpendicular to the optic axis, while the other has a component perpendicular and parallel.



Optical activity corresponds to rotation of linearly polarized light. Material that displays optical activity can be thought of as birefringent for circularly polarized light. Linearly polarized light can be thought of as two circularly polarized beams: one left hand circularly polarized, the other right hand circular polarized. One polarization has one index of refraction, while the other handedness has a different index of refraction. This then results in an angle of rotation $\beta = \frac{\pi d}{\lambda} \; n_L - n_R$. d is the thickness of the material and λ is the wavelength of the incident light.