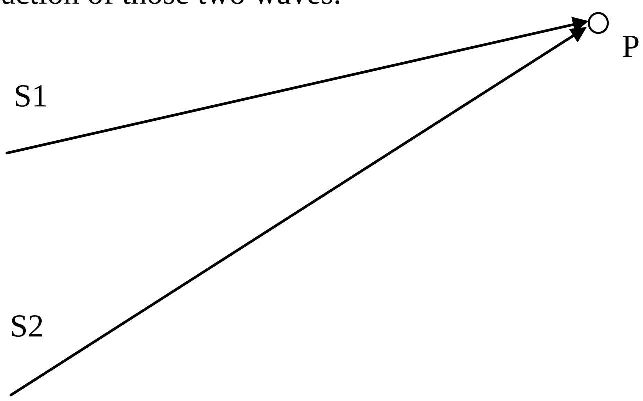
Interference

Imagine that we have two or more waves that interact at a single point. At that point, we are concerned with the interaction of those two waves.



Imagine we have radiation from two plane waves interacting at point P. The radiation from source 1 has the form $\vec{E}_1(\vec{r},t)=\vec{E}_{10}e^{i(\vec{k}_1\cdot\vec{r}-\omega_1 t+\phi_1)}$ and the radiation from source 2 has the form $\vec{E}_2(\vec{r},t)=\vec{E}_{20}e^{i(\vec{k}_2\cdot\vec{r}-\omega_2 t+\phi_2)}$. The vector **r** points from the origin to the point P. What will the net electric field be at point P?

When dealing with light it is important to recognize that we usually consider the irradiance rather than the electric field strength because the electric field varies too quickly for our observation. The irradiance is directly related to the Poynting vector, **S**. The magnitude of the Poynting vector is equal to $S=\varepsilon v\left\langle E^2\right\rangle$ where ε is the dielectric constant of the medium and v is the speed of light in that medium. E is the NET electric field at that location. The < > mean time average. Why do we average the square of the net electric field?

To calculate the instantaneous value of the Poynting vector we must consider $E^2=\vec{E}_{net}\cdot\vec{E}_{net}$. For our particular situation this corresponds to $E^2=(\vec{E}_1+\vec{E}_2)\cdot(\vec{E}_1+\vec{E}_2)=\vec{E}_1\cdot\vec{E}_1+\vec{E}_2\cdot\vec{E}_2+2\vec{E}_1\cdot\vec{E}_2$. What will the first two terms correspond to?

The last term corresponds to all interference effects. If we expand the dot product in terms of the expressions for the electric field at any position or time we find: $\vec{E}_1\cdot\vec{E}_2=\vec{E}_{10}\cdot\vec{E}_{20}e^{i(\vec{k}_1\cdot\vec{r}+\vec{k}_2\cdot\vec{r}-(\omega_1+\omega_2)t+\phi_1+\phi_2)}$. First, let's consider the dot product of the amplitudes. Under what circumstances is it possible to have that dot product be zero?

What does this imply about interference?

We can separate the time dependence into a separate exponential to examine the effect of the time variance: $\vec{E}_1 \cdot \vec{E}_2 = \vec{E}_{10} \cdot \vec{E}_{20} e^{i(\vec{k}_1 \cdot \vec{r} + \vec{k}_2 \cdot \vec{r} + \phi_1 + \phi_2)} e^{-i(\omega_1 + \omega_2)t} = \vec{E}_{10} \cdot \vec{E}_{20} e^{i(\vec{k}_1 \cdot \vec{r} + \vec{k}_2 \cdot \vec{r} + \phi_1 + \phi_2)} e^{-i(\omega_1)t} e^{-i(\omega_2)t}$. If we simply look at the real part of the time dependence we get $\cos(\omega_1 t)\cos(\omega_2 t)$ [this is not 100% correct. I have played loosely with the terms and care must be taken with regards to the real and imaginary parts of both terms, but it is illustrative of the point]. This product can be rewritten as $\frac{1}{2}\left[\cos\left((\omega_1 + \omega_2)t\right) + \cos\left((\omega_1 - \omega_2)t\right)\right]$. This represents the time variance of the interference term. Since we are really interested in the time averaged quantity we have to perform the time average over at least a period. The first term will integrate to zero over a single period or longer. The second term may or may not be zero.

Suppose that the fastest effect we can observe is on the order of a picosecond (10^{-12}s) and that we have two lasers of different wavelength interfering. What would the maximum difference in wavelengths be in order to have a beat period of 1 picosecond?

What does this imply about interference between different wavelength lasers?

Let's make the simplification that the sources are monochromatic, with the same polarization directed out of the page. In this case, the time dependence would simplify to $\cos^2(\omega t)$ which will time average to ½. Therefore the interference term will be $\left\langle \vec{E}_1 \cdot \vec{E}_2 \right\rangle = \frac{1}{2} \vec{E}_{10} \cdot \vec{E}_{20} e^{i(\vec{k}_1 \cdot \vec{r} + \vec{k}_2 \cdot \vec{r} + \phi_1 + \phi_2)}$. The real part of this will be $\left\langle \vec{E}_1 \cdot \vec{E}_2 \right\rangle = \frac{1}{2} \vec{E}_{10} \cdot \vec{E}_{20} \cos(\vec{k}_1 \cdot \vec{r} + \vec{k}_2 \cdot \vec{r} + \phi_1 + \phi_2)$. The argument of the cosine term is referred to as δ which is the phase difference. Therefore we find that if $I_{total} = I_1 + I_2 + I_{12}$, then $I_{12} = \vec{E}_{10} \cdot \vec{E}_{20} \cos(\delta)$. If we assume that E_{10} and E_{20} are parallel, then we can write $E_{10} = \sqrt{2I_1}$ and $E_{20} = \sqrt{2I_2}$. Looking at the total irradiance we find that $I_{total} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$. Clearly the total irradiance will vary with δ and interference problems become one of determining the phase difference introduced both by path difference and initial phase.

Under what circumstances will the total irradiance be a minimum?

Under what conditions will the total irradiance be zero?

How would you change the expression if the light were polarized in the plane of the paper?

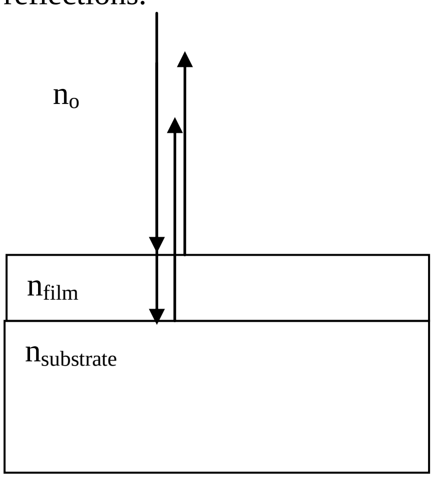
Would it be possible to have interference between two beams of elliptically polarized light? Explain.

We have made a very important assumption in the previous work. What is that assumption?

Thin film interference

Anti Reflection (AR) coating

Consider a single piece of glass. When light is incident upon this piece of glass, there will be a reflection. This reflection is caused by the change in index of refraction. The reflection coefficient for normal incidence is given by $r = \frac{n_1 - n_2}{n_1 + n_2}$. This reflection coefficient also incorporates the phase shift for external and internal reflections.



For an AR coating, what condition must we require for the phase difference of the two reflected beams?

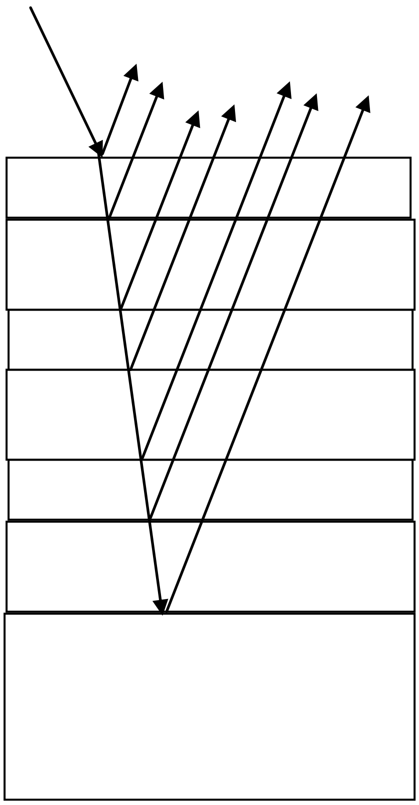
What must be true about the amplitudes of the two reflected beams?

Find a condition on the thickness of the film and its index of refraction so that there is no reflected light. Note that you want to use the minimum thickness possible (why?)

Better AR coatings can be made by stacking several thin films, but the principle is the same.

Dielectric mirrors

Consider a stack of thin films that vary index of refraction between two values n_1 and n_2 with n_2 greater than n_1 or n_o. Consider that each film is a ¼ wavelength thick.



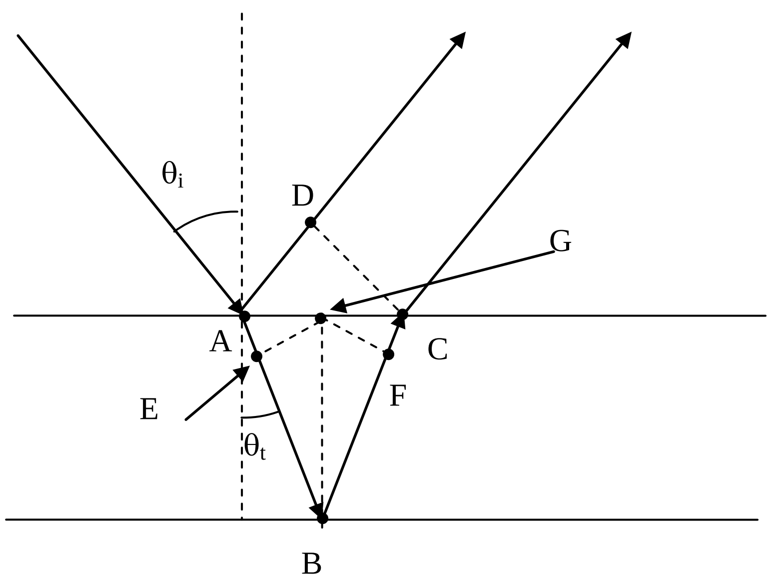
Which film corresponds to the higher index of refraction?

What is the relative phase of each reflecting beam (consider that they are all normal (not as drawn)?

What is the overall reflectivity of this situation at the particular wavelength?

What would happen if you moved off of this "design" wavelength?

Single layer thin film – off normal



Consider the beam of light incident at an angle θ_i upon a film of index of refraction n_f and thickness t. Write an expression for the optical path difference between the reflected rays (that traveled into the film and reflected) and the ray that reflected from the first surface. You may use the letters to denote lengths. Note that point at which both sets are in the same medium again is useful (i.e. use the path difference between the rays going to D and to C). The dotted lines are all perpendicular to either the surface or two a ray.

Now, relate the path length AE=CF to AG and the angle θ_t.

Relate the length AD to AC and the angle θ_i.

Based on these two expressions, relate AE+CF to AD and the indices of refraction.

Now expand the optical path difference replacing AE+CF with your relation from the previous step and simplify.

Relate the length EB=BF to the thickness of the film, index of refraction and the angle of incidence.

Write an expression for the phase difference between these two rays.

Suppose you had a point source located above the film. What would you expect to see in the film? Explain.