WS 26 Poynting Vector and Irradiance

We want to figure out the rate of energy delivery by an electromagnetic wave.

Maxwell's Equations are given below.

1) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$; 2) $\nabla \times \vec{B} = \mu \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}$; 3) $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$; 4) $\nabla \cdot \vec{B} = 0$.

Energy density of electric and magnetic fields can be determined through capacitance and inductance expressions. The net result is that the energy per unit volume stored in the electric field is given by: $u_E = \frac{1}{2}\epsilon_0 E^2$ and the energy density stored in the magnetic field is given by: $u_B = \frac{1}{2}\frac{1}{\mu_0}B^2$. $u_E = u_B$ and therefore $u = 2\sqrt{u_E}\sqrt{u_B}$. What is this expression?

What are the dimensions of ϵ_0 and μ_0 ? What are the dimensions of the product of these two constants? What is the value of $\sqrt{\epsilon_0 \mu_0}$ and its units?

The Lorentz force is given by: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. Consider the work done on a charged particle by an oscillating electric and magnetic field.

Which parts of this force does work on the charged particle? Explain.

Determine the rate of work done on the charged particle $\left(\frac{dW}{dt}\right)$ at any instant.

If we imagine that the charge is spread over some volume, then this becomes

$\int_{vol} \rho \vec{v} \cdot \vec{E} d^3x$, but the charge density multiplied by the velocity is the current density $\vec{J}$. so this is $\int_{vol} \vec{J} \cdot \vec{E} d^3x$. Using Maxwell's equations, substitute for the current density in terms of the magnetic and electric fields.

Use the vector identity: $\nabla \cdot (\vec{E} \times \vec{B}) = -\vec{E} \cdot \nabla \times \vec{B} + \vec{B} \cdot \nabla \times \vec{E}$ to rewrite your new expression and try to get terms of the form of $\vec{B} \cdot \frac{\partial \vec{B}}{\partial t}$ and $\vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$ through judicious use of Maxwell's equations.

Suppose you were taking the partial derivative with respect to time of the electric and magnetic field energy densities. What would those look like?

Noting that the current density of free space is zero, you should find a term of the form $\int \left(\nabla \cdot \left(\vec{E} \times \vec{B}\right) - \frac{\partial u}{\partial t}\right) dV = 0$ with some multiplicative constants. What this term means is that the time rate of change of the energy out of a volume is equal to the spatial rate of change of the quantity proportional to $\vec{E} \times \vec{B}$. This quantity is the Poynting vector and it points in the direction of energy flow.

Calculate the energy in a column of light of length $c\Delta t$ and cross sectional area A using the energy density.

Suppose you wanted the power of this beam of light, what would that be? Relate this to irradiance and the Poynting vector.