WS25 Wave equation for electro-magnetic waves

Maxwell's Equations are given below.

1) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$; 2) $\nabla \times \vec{B} = \mu \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}$; 3) $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$; 4) $\nabla \cdot \vec{B} = 0$. They are restatements of Faraday's law of induction, Ampere's Law, Gauss's law, and no magnetic monopoles, respectively.

Suppose we take the curl of equation 1) $\nabla \times \nabla \times \vec{E} = -\frac{\partial \ \nabla \times \vec{B}}{\partial t}$. Using equations 2) and 3), the vector identity $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$, and the fact that the free charge density ρ and the current density $\vec{J}$ equal zero in free space to find a partial differential equation that describes the electric field. Once you have found that expression, describe what it means about the electric field.

Repeat the process for the magnetic field.

What would be a reasonable form of solution for the equation for the electric and magnetic fields?

Using this solution, substitute into equation 1) for a wave traveling in the + z-direction, with the electric field oriented in the + y direction. In this situation $\nabla \times \vec{A} = -\frac{\partial A_y}{\partial z}\hat{x}$. Use this information to relate the amplitude of the electric field to that of the magnetic field.