

Monochromatic aberrations are more complex and indeed there are several ways to approach them. Consider that we made a great deal of use of the small angle approximation. These approximations were that the

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \quad \text{and} \quad \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

As the angle increases, there is a larger deviation from ideal imaging.

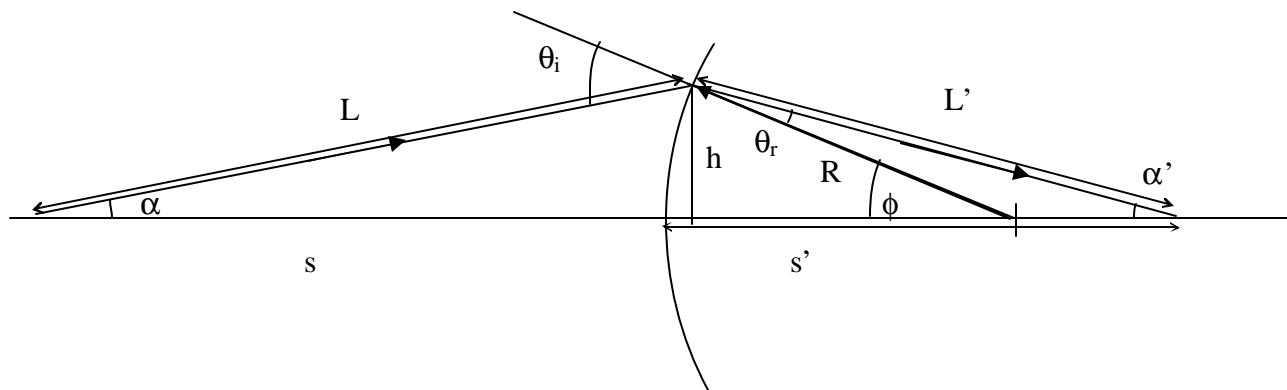
- When we consider an image, there is a \_\_\_\_\_ to \_\_\_\_\_ between a \_\_\_\_\_ on the object and a \_\_\_\_\_ on the \_\_\_\_\_.
- When we talk about a deviation from ideal imaging, what do you think that means?

The next term for the sin is the  $x^3$  term so aberrations accounted for by this term are known as third order aberrations or Seidel Aberrations. These aberrations are **spherical aberration, coma, astigmatism, curvature of field** and **distortion**.

### Spherical aberration

Spherical aberration is the only ‘on-axis’ aberration – that is the points lie upon the optic axis. All other aberrations occur for non-axial points. To understand spherical aberration, consider the curved refracting surface shown below. Previously, we looked at this and made the approximation that  $\alpha$ ,  $\theta$ , and  $\phi$  were small.

This was the paraxial approximation.



However, let's make use of Fermat's principle to see how well our approximation worked. First determine the index of refraction of the medium. Assume that the medium with the source is air.

Next measure the physical path lengths and determine the optical path length (think of this as related to the apparent transit time multiplied by  $c$ ) for each of the paths shown to the image. Compare the two optical path lengths. Based on Fermat's principle, how should these paths be related?

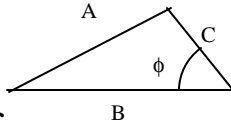
Do your measured/calculated optical path lengths agree with Fermat's principle?

Let's move on to calculating the path lengths. Previously, for a spherical refracting surface we determined the following relation between the object distance, the image distance and the radius of curvature:  $\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{R}$

Calculate the time it takes to reach the image point  $s'$  for an on axis ray and then use that to calculate the optical path length ( $ct$ ).

Write an expression for the optical path length for light traveling along the off axis path ( $L$  and  $L'$ )

Now use the law of cosines to write  $L$  and  $L'$  in terms of  $s$ ,  $s'$ ,  $R$  and  $\phi$  using the law of cosines



$$A^2 = C^2 + B^2 - 2BC \cos \phi$$

Then using the first two orders of the binomial expansion:  $[1+x]^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$  the first three terms of

series expansion of  $\cos(\phi) = 1 - \frac{(\phi)^2}{2!} + \frac{(\phi)^4}{4!}$  and substitute  $\phi = \frac{h}{R}$ , find an order  $h^4$  correct form for  $L$ . Then,

by inspection write a similar expression for  $L'$ .

Now that you have expressions for the OPL's, you should set the OPD to zero (Fermat's principle) and solve

$$\text{for } \frac{n}{s} + \frac{n'}{s'}$$

Off Axis Aberrations

Coma

Astigmatism

Curvature of field

Distortion