Worked Example: What can you assert problem.

A skater is gliding along frictionless ice with a velocity of 4 m/s SE when the wind come up and accelerates her at 1.6 m/s^2 W for 3 s. Then the wind shifts and strengthens accelerating her at 2.4 m/s^2 NE for 5 sec. What can you assert about this situation?

What can you assert problems are actually asking us to carry out a thorough analysis of the given situation to determine everything that can be determined, using relevant principles and relations, from the given information. That means we need to think about what is happening in the situation so that we can determine what physical principles and relations apply.

Thinking about the situation we should realize that the skater was initially traveling with a constant velocity. Then she was accelerated for a period of 3 seconds. During this period her velocity will be changing at a constant rate. So we can calculate her velocity at the end of each of the three seconds and we could calculate her average velocity during each one second interval and during the three second period. Knowing the average velocities we can calculate her displacement during each interval. In all of these calculations we have to keep in mind that velocity and displacement are vector quantities so we will probably want to use components in the two directions N-S and E-W.

We can also calculate the equivalent speeds and distances traveled for the interval. We know that the average speeds and the distances traveled will have different values from the magnitudes of the average velocities and the displacements since the skater is not traveling in a straight line.

After the 3 seconds the wind shifts causing the acceleration to change. We can determine exactly the same quantities for the next 5 second interval since once again the skater experiences a constant acceleration. The magnitude and direction of the acceleration is different in this time interval, but the basic principles are exactly the same.

Finally we can calculate the overall average velocity, average speed, displacement and distance traveled. We will have to be careful in getting the overall average velocity and speed not to simply add the initial and final values and divide by two. That procedure does not apply in this case. Do you know why?

Considering our situation with the skater the first part of the statement tells us that the skater is moving with a constant velocity, that is what the information about frictionless ice means. So initially the skater is going southeast at a constant rate of 4 m/s. Then the wind accelerates her at a rate of 1.6 m/s^2 directed west for 3 seconds. Consequently, she will experience a change of velocity of 1.6 m/s west each second for 3 seconds. Knowing this we are now in a position to determine the skater's velocity at the end of each of the three seconds.

The skater's initial velocity was 4 m/s SE. The two components of this velocity are 2.83 m/s S and 2.83 m/s E because the SE means the angle between her velocity vector and both the South and East axes was 45^o. To find her velocity after the first second we add the change in velocity during that second to her initial velocity. So we add 1.6 m/s West to 2.83 m/s E and 2.83 m/s S. Since West and East are actually oppositely directed the vector addition process becomes a subtraction. That means the skater's velocity one second after the wind starts blowing is 1.23 m/s E and 2.83 m/s S. Continuing in this way the skater's velocity at the end of the second sec is 0.37 m/s W and 2.83 m/s S, and at the end of the third second it is 1.97 m/s W and 2.83 m/s S. Writing this latter velocity as a single vector we have 3.45 m/s 55.1^o S of W.

The skater's average velocity for this time interval is the sum of (2.83 m/s E, 1.97 m/s W) and (2.83 m/s S, 2.83 m/s E) divided by two. In other words, the average velocity is (2.83 m/s S, 0.43 m/s E). Now we are in a position to calculate the displacement for this 3 sec interval. The value is (8.49 m S, 1.29 m E) which is equivalent to 8.59 m 81.7 degrees S of E.

The skater's average speed for the 3 sec time interval is 4 m/s, her initial speed, plus 3.45 m/s, her final speed, divided by 2. The value is 3.73 m/s. So the distance traveled by the skater during this time interval is 11.19 m. Notice that this is larger than the magnitude of the displacement, is such a result reasonable? If yes, why? If no, why not?

Now we carry out exactly the same process for the second time interval of 5 seconds duration. This time our starting velocity is the 3.45 m/s 55.1^o S of W and the acceleration is 2.4 m/s^2 NE. Rewriting these in component form we have 2.83 m/s S and 1.97 m/s W as the components of the velocity and 1.7 m/s^2 N and 1.7 m/s^2 E as the components of the acceleration. The resulting velocities at the end of each second of this time interval are:

Time (s)	1	2	3	4	5
N-S comp	1.13 m/s S	0.57 m/s N	2.27 m/s N	3.97 m/s N	5.67 m/s N
E-W comp	0.27 m/s W	1.43 m/s E	3.13 m/s E	4.83 m/s E	6.53 m/s E

This is equivalent to 8.63 m/s 41^o N of E.

The skater's average velocity for this time interval is (2.83 m/s S, 1.97 m/s W) plus (5.67 m/s N, 6.53 m/s E) divided by two. The value is (1.42 m/s N, 2.28 m/s E), which is equivalent to 2.69 m/s 32^o N of E. Which means the displacement is (7.1 m N, 11.4 m E) and that is equivalent to 13.4 m 32^o N of E.

The skater’s average speed for this time interval is 3.45 m/s plus 8.63 m/s divided by 2. The value is 6.04 m/s. That means the distance traveled is 30.2 m. Once again this is larger than the magnitude of the displacement, should we be worried this time? Why would the skater travel a longer distance than the straight-line distance between her position at the start of the 5 sec interval and her position at the end of the 5 sec interval?

Now we are in a position to determine the average velocity (speed) and the displacement (distance traveled) for the entire motion, i.e., over the 8 sec interval identified.

We have to be careful to take account of the fact that there were two different accelerations during this 8 sec interval. This fact is important because it means we **cannot** simply take the initial velocity (speed) add it to the final velocity (speed) and divide by 2. Why can’t we use this procedure?

The overall displacement is the vector sum of the two separate displacements. The value is (1.39 m S, 12.69 m E), which is equivalent to 12.77 m 6.2^o S of E. That means the average velocity was (0.17 m/s S, 1.6 m/s E), which is equivalent to 1.61 m/s 6.2^o S of E.

The overall distance traveled was 30.2 m + 11.19 m = 41.39 m. So the overall average speed was 5.17 m/s.

At this point we have found essentially everything worth finding our about this situation.