Worked Example: Working Backwards Problem

An object moves along a horizontal surface in a manner described by the two kinematics equations below. These equations cover the complete time the object is in motion. Solve the equations for the unknowns, construct a motion diagram for the situation, then construct a physical situation that is consistent with the motion diagram and equations, and finally draw the position—time and velocity—time graphs for the motion.

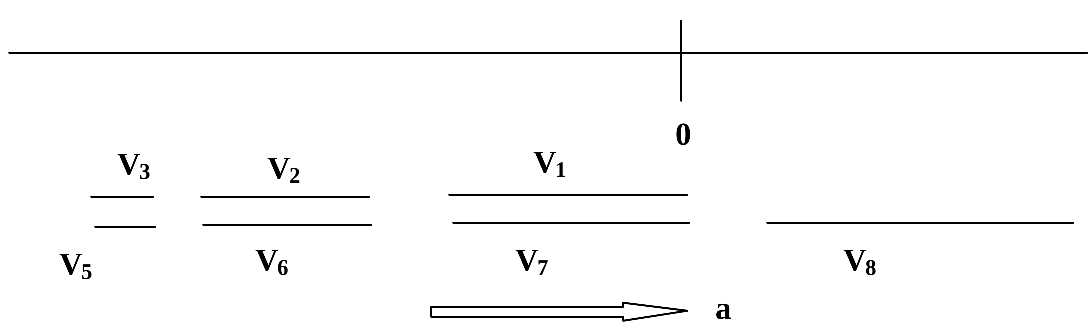
$$x_1 = 0\text{ m} + (-4\text{ m/s})(1.53\text{ s}) + (0.5)(a)(1.53\text{ s})^2$$

$$4.9\text{ m} = x_1 + (0.5)(a)(2.47\text{ s})^2$$

In a standard Working Backwards problem we are presented with the equations describing the situation, which are normally the next to last thing found in working a problem, and required to "work backwards" to a physical situation that could have produced the equations. That means we must realize that the equations are a model of a physical situation, decipher those equations to determine what is happening, and then construct a reasonable physical situation that could have produced the equations. It is often useful and/or important to keep in mind that there may be more than one solution to the problem.

Looking at the two equations in this problem we see that they are both specific examples of the general relation: $x_f = x_o + v_o t + 1/2at^2$. With this recognition we can say that in the first time interval, i.e., the 1.53 s interval, the object started at the origin with an initial velocity of 4 m/s in the negative direction and had a positive acceleration throughout the interval. For the second time interval, i.e., the 2.47 s interval, the object ends up at the position + 4.9 m having started with no initial velocity for the interval, but having a positive acceleration throughout. The fact that the same symbol is used for the acceleration in both equations means that the same acceleration applies throughout the full 4.0 seconds covered by the equations.

At this point it is useful to translate our analysis into a motion diagram. It will look like:



The line at the top of the motion diagram, with the 0 on it, defines our reference frame. As you can see to the right is the positive direction for our diagram. The velocity vectors in the motion diagram are labeled with different subscripts since the velocity is constantly changing during the two intervals. There is also a gap in the numbering of the velocities since the object stops, **AT AN INSTANT,** between when $\mathbf{V_3}$ **and** $\mathbf{V_5}$ occur. There is only one arrow representing the acceleration and it points to the right since the acceleration is positive and constant for the full 4.0 s interval.

Since we have two equations in two unknowns we can solve the equations to find the acceleration and the position 1.53 s into the interval. The values we get are 2.61 m/s^2 and -3.06 m respectively.

In terms of constructing a physical situation we can identify the basic character of the motion from the motion diagram. We have an object that is initially moving in the negative direction at 4 m/s when it is subjected to a positive acceleration for a period of 4 s. There is a large variety of physical systems that could satisfy the equations, all we need is one appropriate example.

One possibility is a toy car initially moving up an incline. The car is moving at 4 m/s at the instant it passes the point designated as the origin for the reference frame and is first timed. The positive direction is parallel to the incline surface and down. The car continues up the incline for 1.53 s, stops at an instant, and then accelerates down the incline passing the point 4.9 m down the incline from the origin at the end of the period timed, i.e., 4 s.

The sketches of the position-time and velocity-time graphs are shown below.