Making Real-World Connections With Your TI-84 Plus CE Graphing Calculator

35th Annual T³ International Conference
Fort Worth, Texas

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10:30 a.m. – 11:50 a.m.
Omni Hotel Level 2
Fort Worth Ballroom 6

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Designing a Water Fountain

We will use quadrati c functions to design a water fountain and investigate how beauty lies in not just the motion, but in the formulas of the functions themselves. The fountain below is in New York LaGuardia Airport Terminal B. Their water presentations pair together light, motion, music, and, of course, the pièce de résistance, the parabola.
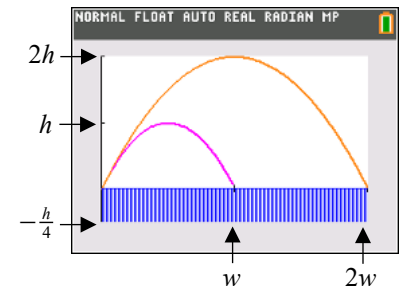

See https://www.youtube.com/@laguardiaterminalb6208/videos for videos of their fountain displays.

1. Press $\boxed{\text{mode}}$ and select Simultaneous Graphing.
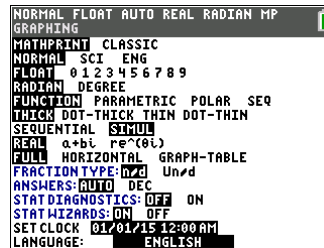
2. We will create formulas of parabolas to design a water fountain similar to the one shown with these specifications:

	Maximum height	Horizontal Range
Small parabola	h	w
Large parabola:	2h	2w

There are many possible formulas, based on what you choose for h and w. The value of h can be different than the value of w.

Once you decide h and w, one possible design is to set your viewing window to $0 \le x \le w$ by $-\frac{h}{4} \le y \le h$.

Optional (TI-84CE only): If desired, we can add the shading to represent the blue pool once we have the formulas.

Work in your group to create at least three different pairs of formulas in **factored form**.
See the next page for a hint.

Hint: First write and graph parabolas in factored form with a zero of w and a zero of $2w$.
Press $\boxed{\text{trace}}$ to check the maximum height of the small parabola.
If needed, multiply the formula of the small parabola by an appropriate scale factor
to adjust its maximum height so it is a height of h. Using the table feature may expedite your work.
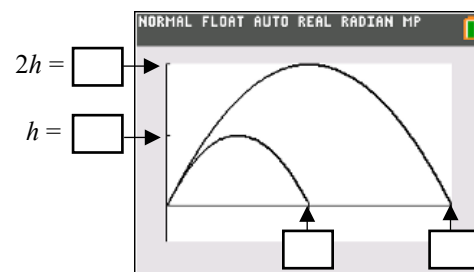
Complete the boxes for the three parabolas.

2h = $\boxed{}$ 2h = $\boxed{}$ 2h = $\boxed{}$

h = $\boxed{}$ h = $\boxed{}$ h = $\boxed{}$
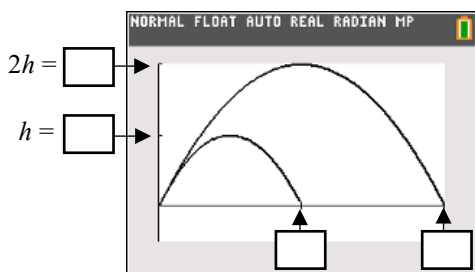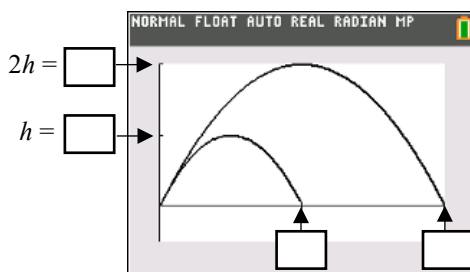
Small Parabola: $y = \boxed{} \cdot x \cdot \left(\boxed{} - x \right)$ Small Parabola: $y = \boxed{} \cdot x \cdot \left(\boxed{} - x \right)$ Small Parabola: $y = \boxed{} \cdot x \cdot \left(\boxed{} - x \right)$

Large Parabola: $y = \boxed{} \cdot x \cdot \left(\boxed{} - x \right)$ Large Parabola: $y = \boxed{} \cdot x \cdot \left(\boxed{} - x \right)$ Large Parabola: $y = \boxed{} \cdot x \cdot \left(\boxed{} - x \right)$

3. Distribute each of your formulas to write these in expanded form. Then complete the boxes.

Small Parabola: $y = \boxed{} \cdot x - \left(\boxed{} \right) x^2$ Small Parabola: $y = \boxed{} \cdot x - \left(\boxed{} \right) x^2$ Small Parabola: $y = \boxed{} \cdot x - \left(\boxed{} \right) x^2$

Large Parabola: $y = \boxed{} \cdot x - \left(\boxed{} \right) x^2$ Large Parabola: $y = \boxed{} \cdot x - \left(\boxed{} \right) x^2$ Large Parabola: $y = \boxed{} \cdot x - \left(\boxed{} \right) x^2$
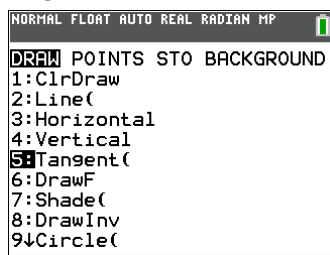
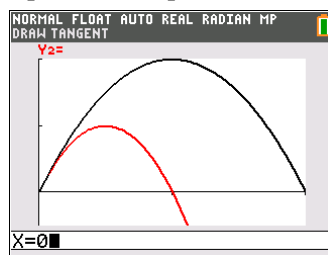**Pattern Sniffing**: What patterns do you notice when looking at all of your formulas?

4. The initial angle of inclination of the fountain jet is related to the slope of the **tangent line**.
   Informally, if a midnight train were traveling on the path of the curve, the locomotive headlight would
   travel in the direction of the tangent line to the curve. Complete the steps below to find its slope.
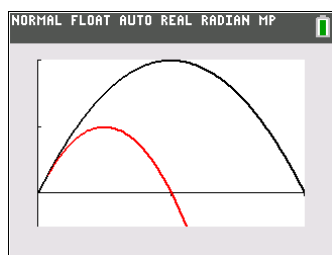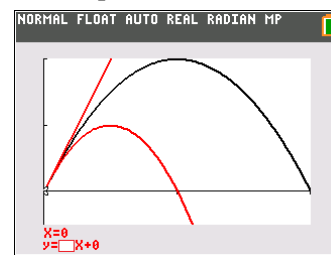
1) Press $\boxed{\text{graph}}$.   2) Press $\boxed{\text{2nd}}$ **[draw]**   4) Type 0 for the x value   5) Press $\boxed{\text{enter}}$.
3) Select **5:Tangent(**   of the point of tangency

a. Notice the slope of this tangent line for your pair of parabolas. What do you observe?

b. Compare what you observed with your pair of formulas in expanded form in Question 3.
   Discuss with your group. Is *their* observation the same for *their* pair of parabolas?
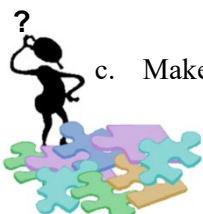
c. Make a conjecture: In general, if a parabola is written in the form $y = ax^2 + bx$,
   the slope of the line tangent to the graph at the origin
   through the y-intercept of the parabola is _____ .

3

Teacher Notes: Optional Flourishes

Sit your cursor on the = sign and press [enter] to deselect Y2. Put cursor on Y3, press [◄] [enter], then change the color.

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3=
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2◻12X(1-X)
■\Y3=
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\
■\    Color:  RED
■     Line:   \
■\       OK   CLEAR
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3=■
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

Put cursor on Y3, press [math] **B:piecewise(**, select 1 piece, press [alpha] [f4] (above [trace]), and select Y2 as shown:

```
NORMAL FLOAT AUTO REAL RADIAN MP
MATH NUM CMPLX PROB FRAC
5↑*√
6:fMin(
7:fMax(
8:nDeriv(
9:fnInt(
0:summation Σ(
A:logBASE(
B:piecewise(
C:Numeric Solver…
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
MATH NUM CMPLX PROB FRAC
5↑*√
6:    Pieces:1
7:
8:       OK   CLEAR
9:fnInt(
0:summation Σ(
A:logBASE(
B:piecewise(
C:Numeric Solver…
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{■;⠿
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{⠿;⠿
■\Y4=
■\Y5=              1:Y1 6:Y6
■\Y6=              2:Y2 7:Y7
■\Y7=              3:Y3 8:Y8
■\Y8=              4:Y4 9:Y9
                   5:Y5 0:Y0
 FRAC FUNC MTRX YVAR
```

Press [2nd] [test], press [►] [►] to get to the CONDITIONS menu, select **1: X ≤**, and type the right endpoint:

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;■
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
TEST LOGIC CONDITIONS
1:X≤
2:X<
3:X≥
4:X>
5:≤X and X≤
6:<X and X<
7:≤X and X<
8:<X and X≤
9↓X=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1⠿
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```


Use Y4 to graph the line *y* = 0. Select the spinner to **change the line style to shade below blue.** ◣

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1
■\Y4◻0
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1
◣\Y4◻0
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\
■\    Color:  BLUE
■     Line:   ◣
■\       OK   CLEAR
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```


On Y5, press [2nd] [rcl] Y3, enter *y* = 24*x*(0.5 − *x*) or *y* = 12*x* − 24*x*^{2} (for *this* set of curves) with the same initial slope.

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1
◣\Y4◻0
■\Y5=■
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1
◣\Y4◻0
■\Y5=
■\Y6=
■\Y7=
■\Y8=
Rcl
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1
◣\Y4◻0
■\Y5=              1:Y1 6:Y6
■\Y6=              2:Y2 7:Y7
■\Y7=              3:Y3 8:Y8
■\Y8=              4:Y4 9:Y9
                   5:Y5 0:Y0
 FRAC FUNC MTRX YVAR
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1
◣\Y4◻0
■\Y5=
■\Y6=
■\Y7=
■\Y8=
Rcl Y3
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1
◣\Y4◻0
■\Y5◻{■2;X≤1
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
■\Y1◻6X(2-X)
■\Y2=12X(1-X)
■\Y3◻{Y2;X≤1
◣\Y4◻0
■\Y5◻{24X(.5-X);X≤.5
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

Possible Extensions:
1. Create formulas to construct the fountains below. All have the same initial slope at the water's surface. What do you notice? What do you wonder?
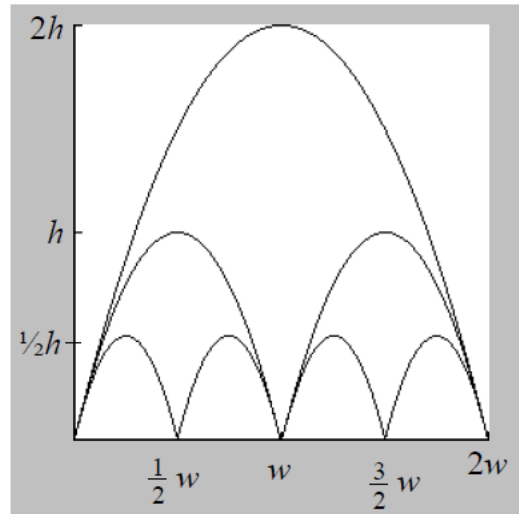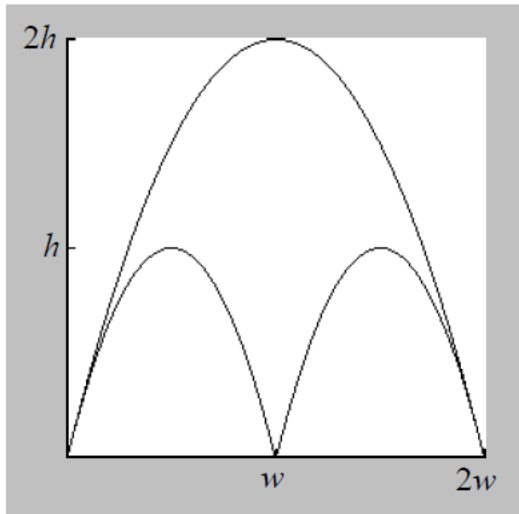
2. Explore the two fountains shown and create formulas for their design. For parabolas with the same initial slope at the water's surface, if the horizontal range w of the smaller parabola is 2/3 the horizontal range of the larger parabola, is the maximum height h of the smaller parabola 2/3 the maximum height of the larger parabola?
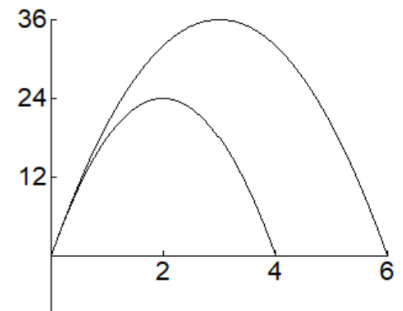
Same question for the ratio of 1/3.
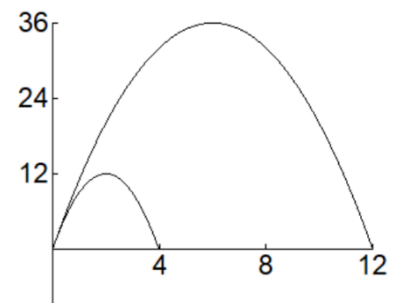
Generalize this question to any parabola and any ratio.
Hint 1: Consider $y = ax(b - x)$ and $y = bx(a - x)$ which have the same initial slope ab.
Hint 2: To find the coordinates of the vertex of a parabola, look midway between its zeros.

3. Create your own water fountain design using as many parabolas as you wish.
   They should have the same angle of inclination at the origin.
   Be sure to supply your equations and your viewing window.

Sample possible answers to the student investigation are shown.

$2h =$ **1**	$2h =$ **18**	$2h =$ **80**
$h =$ **.5**	$h =$ **9**	$h =$ **40**
1 **2**	**3** **6**	**4** **8**
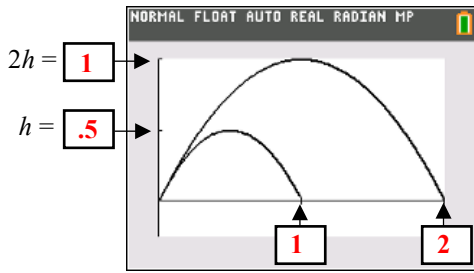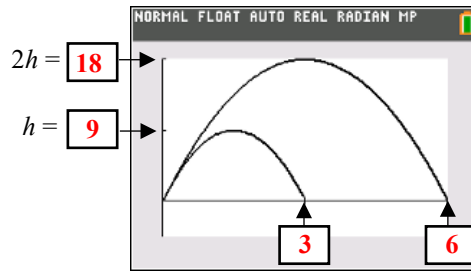
Small Parabola: $y = \boxed{2} \cdot x \cdot (\boxed{1} - x)$ Small Parabola: $y = \boxed{4} \cdot x \cdot (\boxed{3} - x)$ Small Parabola: $y = \boxed{10} \cdot x \cdot (\boxed{4} - x)$

Large Parabola: $y = \boxed{1} \cdot x \cdot (\boxed{2} - x)$ Large Parabola: $y = \boxed{2} \cdot x \cdot (\boxed{6} - x)$ Large Parabola: $y = \boxed{5} \cdot x \cdot (\boxed{8} - x)$

Small Parabola: $y = \boxed{2} \cdot x - (\boxed{2}) x^2$ Small Parabola: $y = \boxed{12} \cdot x - (\boxed{4}) x^2$ Small Parabola: $y = \boxed{40} \cdot x - (\boxed{10}) x^2$

Large Parabola: $y = \boxed{2} \cdot x - (\boxed{1}) x^2$ Large Parabola: $y = \boxed{12} \cdot x - (\boxed{2}) x^2$ Large Parabola: $y = \boxed{40} \cdot x - (\boxed{5}) x^2$

Question 3 asks students to sniff out patterns.
They may notice some of the relationships below, although they likely won't articulate them this formally:

- Comparing *factored* forms, in general, if the small parabola has the factored formula $y = 2ax(w - x)$
  then the larger parabola has the factored formula $y = ax(2w - x)$
  - The *first box for the small parabola* is twice the value of the *first box of the large parabola.*
  - The *second box for the small parabola* is half the value of the *first box of the large parabola.*
  - The *product of the first and second boxes of the small parabola* is the *product of the first and second boxes of the large parabola*. This product is the slope of the curve at the water's surface.
- Comparing *expanded* forms, in general, if the small parabola has the formula $y = 2ax(w - x) = 2awx - 2ax^2$,
  then the larger parabola has the formula $y = ax(2w - x) = 2awx - ax^2$.
  - The *linear coefficient of the small parabola* is the same as the *linear coefficient of the large parabola*, and is the slope of the curve at the water's surface. This is the first box in both formulas for the expanded form.
  - The *leading coefficient of the small parabola* is twice the *leading coefficient of the large parabola*. Observe the second box in both formulas for the expanded form.
- By construction, of the 2 to 1 ratio, this relationship holds:
  The small parabola $y = 2ax(w - x) = 2awx - 2ax^2$ with leading coefficient $-2a$ has a max at $(\frac{1}{2}w, \frac{1}{2}aw^2)$.
  The large parabola $y = ax(2w - x) = 2awx - ax^2$ with leading coefficient $-a$ is has a max at (w, aw^2).

Question 4c asks students to conjecture and complete the blank:
  In general, if a parabola is written in the form $y = ax^2 + bx$,
    the slope of the line tangent to the graph at the origin through the y-intercept of the parabola is __**b**__ .

The conjecture is true for any parabola $f(x) = ax^2 + bx + c$, but requires calculus to show $f'(x) = 2ax + b$ so $f'(0) = b$.

Because this investigation integrates Science, Technology, Engineering, Art, and Mathematics (STEAM), it is sure to make a splash in the classroom. It can also provide an opportunity to support students with the NCTM Mathematical Teaching Practices. You can adapt this for use in Intermediate Algebra, College Algebra, or Precalculus (for example, using the inverse tangent to find the angle of inclination at the water's surface) or Calculus. Answers to the extensions are happily provided upon request. Email the author at lamaster@pfw.edu or john.lamaster@gmail.com.

Using Parametric Equations to Simulate Motion Paths

After doing the prior investigation, some students (and teachers) are often interested in simulating a fountain which has the smaller parabolas spraying in opposite directions as shown to the right.
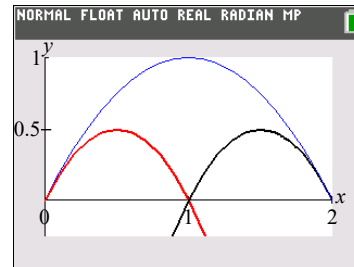
To create this simulation, we use Parametric Graphing Mode.
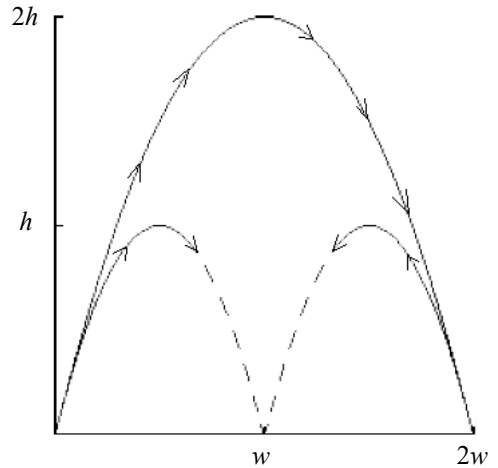
1. Press $\boxed{\text{mode}}$ and select Parametric Graphing.
   Make sure you have also selected Simultaneous Graphing.
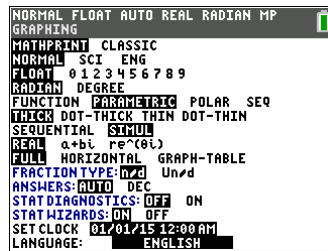
2. In terms of y and x, one possible set of parabolas is as follows:
       Small left parabola: $y = 2x(1 - x)$
       Small right parabola: $y = 2(2 - x)(1 - x)$
       Large parabola: $y = x(2 - x)$

   We can parametrize the large parabola as follows:
   $x = t$
   $y = t(2 - t)$
   Notice by substitution,
   when t is replaced by x in $y = t(2 - t)$,
   we have the parabola $y = x(2 - x)$.

   Optional: Use the spinner to change the graph style to animated path ($\oplus$).

   Similarly, for the small left parabola
       $x = t$
       $y = 2t(1 - t)$
   will produce the parabola $y = 2x(1 - x)$.

   Similarly, for the small right parabola,
   Notice the y-values are the same as
   the outputs of the small left parabola,
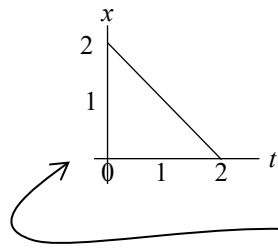   which are $y = 2t(1 - t)$. So we have
       $x = ??$
       $y = 2t(1 - t)$
   where the x-values of the small right parabola decrease linearly from $x = 2$ to $x = 1$ as t increases from 0 to 1.
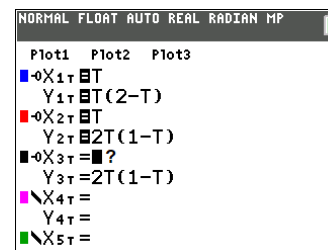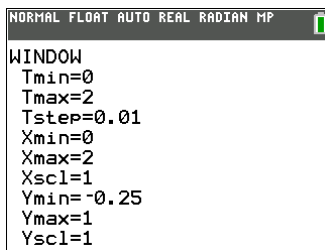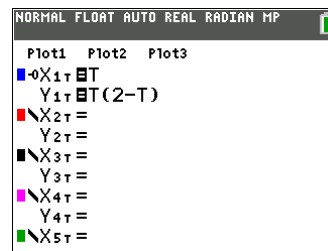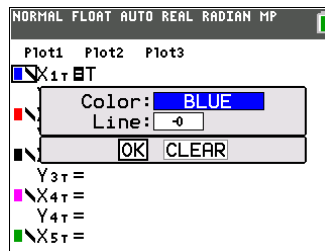
If the *x*-values of the small right parabola decrease linearly from $x = 2$ to $x = 1$ as *t* increases from 0 to 1, we can see by a rough sketch of *x* vs *t* that the formula for the graph of *x* vs *t* is $x = 2 - t$.

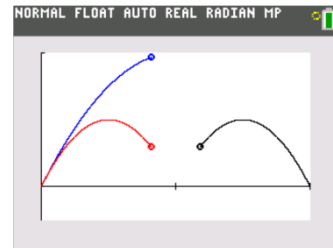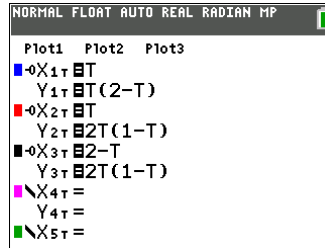Thus the small right parabola can be parametrized as
$$x = 2 - t$$
$$y = 2t(1 - t)$$
We can check: if we solve for *t* in terms of *x*, we have $t = \mathbf{2 - x}$, Now substitute into the equation for $y = 2t(1 - t)$:
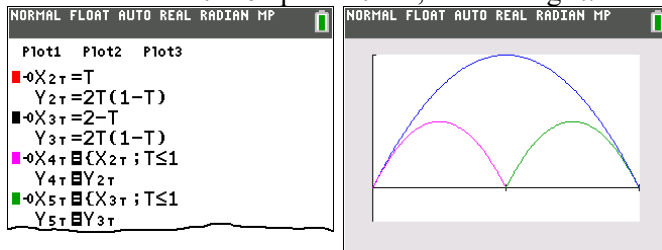$$y = 2(\quad \mathbf{t} \quad)(1 - \quad \mathbf{t} \quad)$$
$$y = 2\mathbf{(2 - x)}(1 - \mathbf{(2 - x)})$$
$$y = 2(2 - x)(x - 1)$$

Optional:
To sketch a piecewise curve, place your cursor on X4, press [math] **B:piecewise(**, select 1 piece, press [alpha] [f4] (above [trace]), and select X2 as shown. We must restrict *t* on the interval from $t = 0$ to $t = 1$. Similarly, for X5, we restrict *t* on the interval from $t = 0$ up to to $t = 1$, even though *x* decreases from 2 down to 0, as in the above linear graph.

While in function mode we can select a spinner to change the line style to shade below blue. ▙, this is not available for parametric graphing. However, if we wish, we can use the shade command in the draw menu to make a pool of water.

Press [2nd] [draw] and sit your cursor on the menu item **7: Shade(** but do not press [enter]. Instead, press [+].

This brings up the Catalog Help.
Memory Tip: "On the 84 Plus,
for help
press [+]"

Use 1 for the pattern and the patres (pattern resolution). For the color#, press [vars], press [▶][▶] to get to the COLOR menu, select **1: BLUE** and type [)] [enter] to paste the command on the on the home screen.
Press [enter] again to run the command and create the blue shading for the pool.
To replay the graph, press [2nd] [draw] **1:ClrDraw**.

To replay the shade command, use [▲] to scroll the Home screen history of entries to select it, press [enter] to paste it on the entry line, and press [enter] again to run the command.   8
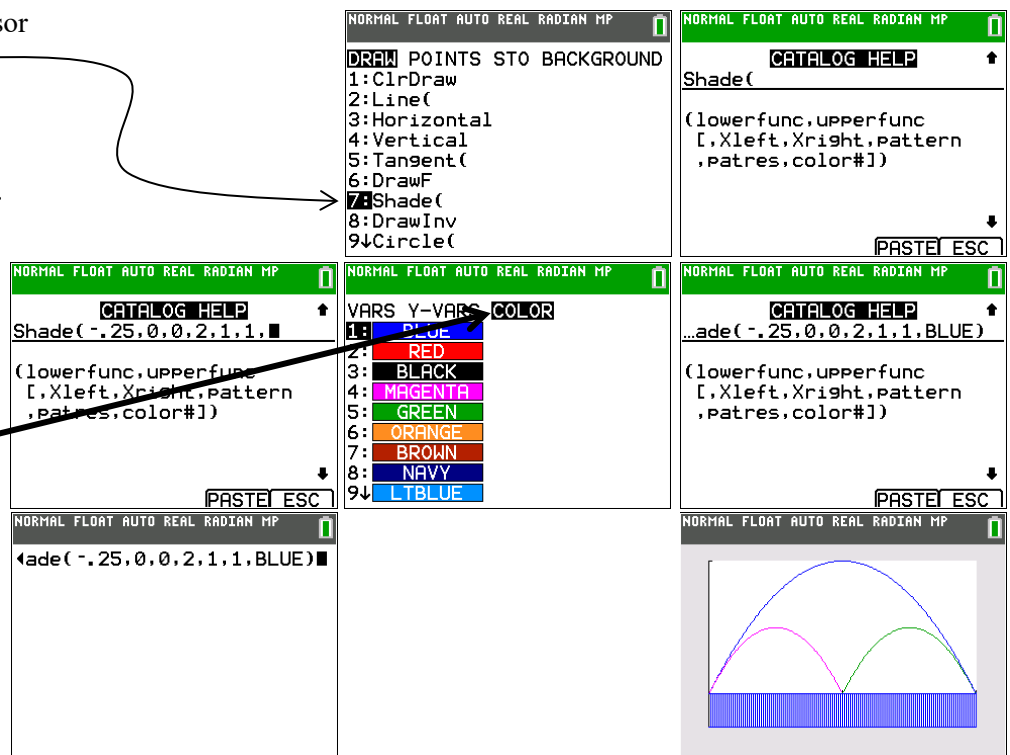
Wile E. Coyote's Roller Skate Chase
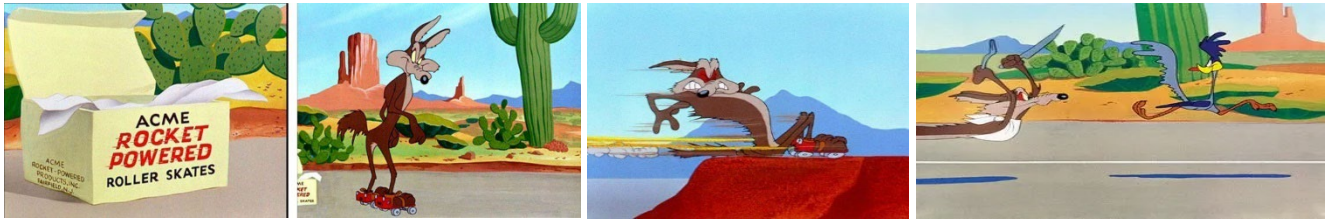
Click on the image to view an clip of this episode.

With the help of some Acme rocket powered roller skates, Wile E. Cote will attempt to snatch the Road Runner. We will use the TI-84 Plus CE and parametric mode to simulate this real-world scenario.

1. Press [mode] and select **Parametric** and **Simultaneous Graphing**. Please make sure you are also in **Radians**.
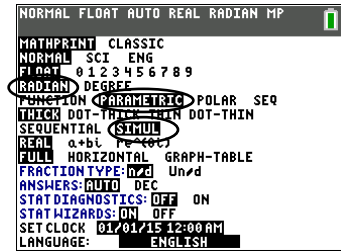
2. Press [zoom] 4: ZStandard for the standard window.
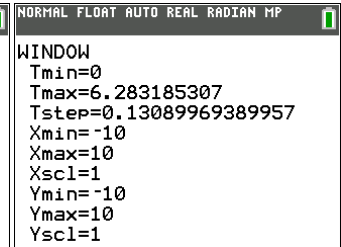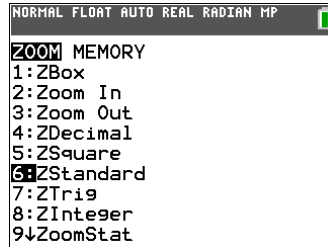
   Press [window].
   Notice the standard settings are
   $0 \le t \le 2\pi$ in steps of Tstep = $\frac{\pi}{24}$
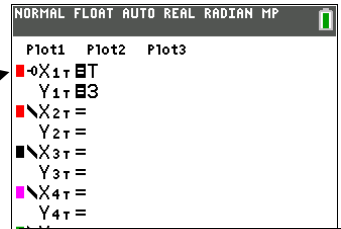   with $-10 \le x \le 10$ by $-10 \le y \le 10$.

3. Press [y=].
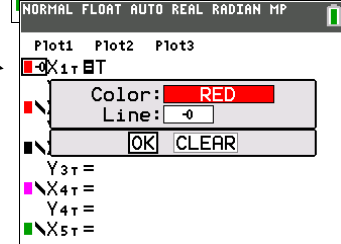   Enter $x(t) = t$, $y(t) = 3$ which will animate the Road Runner's path.
   Press [◄] until you can't any more, then press [enter].
   Optional: Press [►] (84CE) or [enter] (84) to change the graph style to animated path (⊸).

   Optional (TI-84CE only): Use the spinner to change the graph color to red.
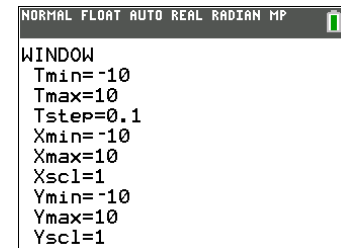
   Press [graph]. Press [trace] and [►] to observe the graph.

4. Press [window] to change the window settings for $-10 \le t \le 10$ so the Road Runner traverses the *entire screen* for $-10 \le x \le 10$.

   TI-84CE: Change Tstep to 0.1 for convenience. Season to taste.
   TI-84: Change Tstep to 0.5 for convenience. Season to taste.
   This affects the speed of the graph.
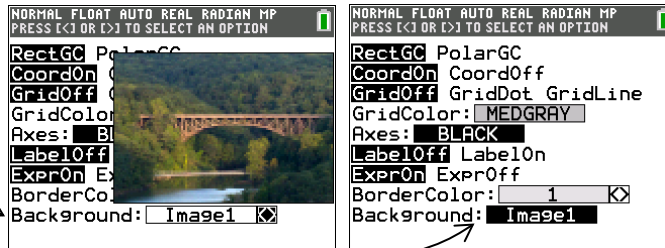
   Press [graph] to see Road Runner cross the screen.

   TIP: To replay the graph, press [2nd][draw] **1:ClrDraw**. Press [graph].

5. Optional (TI-84CE only):
    Press 2nd [format] to access the Graph Format menu.
    Place your cursor on the Background Spinner.
    Press ▶ on the spinner to select Image1.

    Then press ▲ or ▼.
    Image1 should be highlighted.

    NORMAL FLOAT AUTO REAL RADIAN MP
    PRESS [<] OR [>] TO SELECT AN OPTION
    RectGC PolarGC
    CoordOn CoordOff
    GridOff GridDot GridLine
    GridColor
    Axes: B
    LabelOff
    ExprOn E
    BorderCo
    Background: Image1

    NORMAL FLOAT AUTO REAL RADIAN MP
    PRESS [<] OR [>] TO SELECT AN OPTION
    RectGC PolarGC
    CoordOn CoordOff
    GridOff GridDot GridLine
    GridColor: MEDGRAY
    Axes: BLACK
    LabelOff LabelOn
    ExprOn ExprOff
    BorderColor: 1
    Background: Image1

    Press [graph] to see Road Runner cross the bridge.

    NORMAL FLOAT AUTO REAL RADIAN MP
    Plot1 Plot2 Plot3
    ■∘X₁ᴛ=T
      Y₁ᴛ=3
    ■∘X₂ᴛ=2T−9
      Y₂ᴛ=3
    ■\X₃ᴛ=
      Y₃ᴛ=
    ■\X₄ᴛ=
      Y₄ᴛ=
    ■\X₅ᴛ=

6. Press [y=].
    Enter $x(t) = 2t − 9$, $y(t) = 3$ to animate Wile E. Coyote's path.
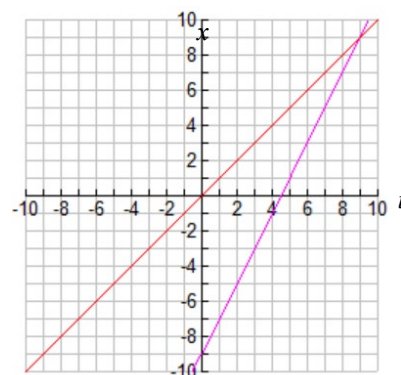   Press 2nd [draw] **1:ClrDraw**. Press [graph].
    What do you notice? What do you wonder?
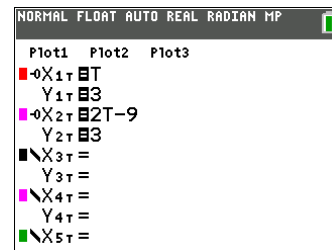
7. To the right are the graphs of $x(t) = t$ and $x(t) = 2t – 9$.
    Notice x is on the **vertical** axis.
    What connections are there between the graphs to the right
    and the animation of Road Runner and Coyote?

8. Work with a partner with the same calculator model and graph style settings as yours.
   Clear or deselect Wile E. Coyote's previous formulas graph in $x2$ $y2$. Describe how does he move for each.
    Sketch a graph of $x(t)$ for each below. What symmetry do you notice in the graphs and the speeds?
    Partner A enters: $x(t) = e^t – 10$ Partner B enters: $x(t) = e^{-t} – 10$
                              $y(t) = 3$ $y(t) = 3$
    Description of how he moves (left, right, speeds up slows down):   Description of how he moves (left, right, speeds up slows down):

    $x = e^t – 10$

    $x = e^{-t} – 10$
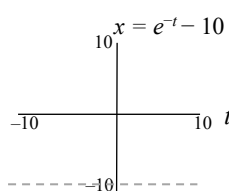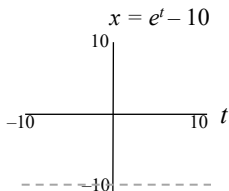
9. On your grapher pick two of the parametric equations below. Assign one to Road Runner and another for Wile and
    enter them both into your grapher, using the same window as before (and the same graph styles for each). Observe
    how each moves across the bridge. Notice when each speeds up and slows down and which one overtakes the
    other. Which formulas go back and forth? Make connections with the graph of $x(t)$ vs t.
    Optional: Construct your own set with $-10 \le x(t) \le 10$ for $-10 \le t \le 10$.

    $x(t) = 10 – 2|t|$ $x(t) = 10 – 0.2t^2$ $x(t) = 0.01t^3$
    $y(t) = 3$ $y(t) = 3$ $y(t) = 3$

    $x(t) = 10\cos(0.1\pi t)$ $x(t) = 10\sin(0.1\pi t)$ $x(t) = \tan\left(\frac{\pi t}{20}\right)$
    $y(t) = 3$ $y(t) = 3$ $y(t) = 3$

    ACME

    &%$@!

    Imagine the graph of the function $x(t)$ vs t is
    crushed by an Acme Anvil and is flattened
    onto the horizontal line $y = 3$.

10

Wile E. Coyote's Flying Wing Suit

With the help of a handy Acme flying wing suit and some bird seed, Wile E. Cote will attempt to drop down from the sky to snatch the distracted Road Runner.

Click on the image to see the animation.

$x = t^{51}$ $y = 3$

1. We will use the parametric equations $x(t) = t^{51}$, $y(t) = 3$ for the Road Runner's path. Before entering these into your grapher, discuss the following with your group.
   a. Describe the behavior of the graph of $x(t) = t^{51}$, shown to the right, for $-1 < t < 1$. What do you notice about his speed?

   b. Describe the behavior of the graph of $x(t)$, shown to the right, for $t < -1$ and $t < -1$. What do you notice about his speed?

   c. At $t = 0$ Road Runner is at position x _____ $y =$ _____ .
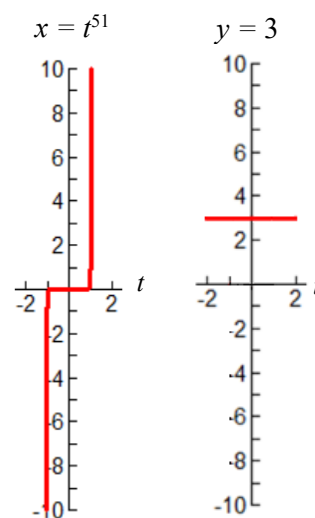
2. We will use the parametric equations $x(t) = e^{-t}$, $y(t) = e^{-t} + 3$ for Wile E. Coyote's path. Before entering these into your grapher, discuss the following with your group.

   $x = e^{-t}$ $y = e^{-t} + 3$

   a. i. Eliminate the parameter to report the path in terms of y and x.

      $y(x) =$ _____ (This is called an *explicit* form.)
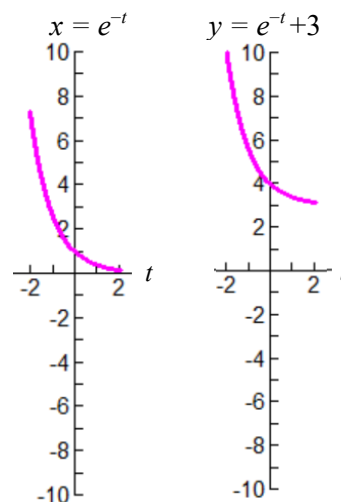
      ii. What kind of function is this?_____
      iii. Complete: As x increases, the graph of $y(x)$ _____.
           {increases, decreases}

           As t increases, the graph of $y(x)$ _____.
           {increases, decreases}
   b. On $-2 \le t \le 2$, does Wile E. Coyote ever reach Roadrunner?   YES NO

3. Press $\boxed{Y=}$, enter the formulas into your grapher. Press $\boxed{window}$ to change the window settings to simulate this from $-2 \le t \le 2$ with Tstep = 0.01. Press $\boxed{graph}$.

4. Finale: The parametric equations $x = -\sqrt{t}$, $y = -4.9t + 3$ simulate the image to the right. Eliminate the parameter to report the path explicitly in terms of y and x.
   $$y(x) = \text{_____} , \quad x \le 0$$

Click on the image to see the animation.

11

Teacher Notes for Wile Coyote and the Road Runner

A common theme for these activities is to ask students to create a scenario and then use mathematics as a building tool, along with the TI-84 Plus calculator. Mathematics becomes the means to the end rather than the end itself. These activities can be done in a mixed calculator classroom with both TI-84's and TI-84CE's, although the latter is much more engaging.

Wile E. Coyote's Roller Skate Chase

For **Question 6**, students may notice Wile's speed is twice the speed of the Road Runner and may wonder when he overtakes him. Both are on cruise control, traveling at a constant speed.

For **Question 7**, students may notice the graph of Wile is twice as steep as the graph of the Road Runner.

In the simulation we can't see Wile at all on $-10 < t < 0.5$; he first appears at $t = 0.5$ (solve $2t - 9 = -10$) and will overtake Road Runner at $t = 9$ (solve $2t - 9 = t$).

The objective of **Question 8** is for students to visualize the dramatic change in speed of the exponential functions (foreshadowing the next activity) and see that the graph of $x(t) = e^t - 10$ is a horizontal reflection of $x(t) = e^{-t} - 10$, which corresponds to the behavior on the simulation. The graphs can be rough sketches done without technology.

$x(t) = e^t - 10$ $x(t) = e^{-t} - 10$
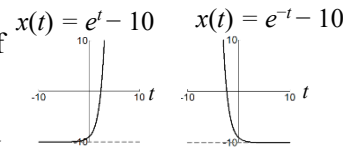
Question 9 helps students with a skill they will later need when using L'Hopital's Rule or Comparison Tests for Convergence: namely, visualizing the end behavior of a function and which one surpasses the other. enables students to experience these functions with new eyes. Based on the level of your students, you could supply less formulas and ask/help students to create their own.

Wile E. Coyote's Flying Wing Suit

For **Questions 1 and 2**, students are given the graphs of the parametric equations for the Road Runner and Wile's motion paths and asked to predict what to expect *before* they run the simulation. For **Question 1**, the rate of change is dramatically different, which is why the graph of $x(t)$ vs t "appears" to be perfectly horizontal on $-1 < t < 1$ and perfectly vertical otherwise, even though it is not. Road Runner will speed up for $t < -1$, slow down abruptly for $-1 < t < 1$ (perhaps to eat birdseed) and rapidly speed up again for $t > 1$, moving across the screen to the right. For Question 2, the parametric equations $x(t) = e^{-t}$, $y(t) = e^{-t} + 3$ for Wile E. Coyote's motion path on the interval $-2 \le t \le 2$ has the explicit formula $y(x) = x + 3$, a linear function with slope 1 and y-intercept $(0, 3)$. As x increases, the graph of $y(x)$ increases (since the slope is positive), but as t increases, the graph of $y(x)$ decreases (since $y(t)$ is a decreasing exponential) Wile E. Coyote's path is the graph of the line $y = x + 3$, which increases as x increases; however, Wile E. Coyote's x and y will decrease as t increases, since $x(t)$ and $y(t)$ are decreasing exponential functions, and more and more slowly. On $-2 \le t \le 2$, Wile E. Coyote will theoretically never reach the point $x = 0$, $y = 3$.

For the Finale

Prior to the class, enter $x = -\sqrt{t}$, $y = -4.9t + 3$ in your grapher in X_{6T}, Y_{6T} as shown.
Deselect the equations by sitting your cursor on the = sign and pressing enter.
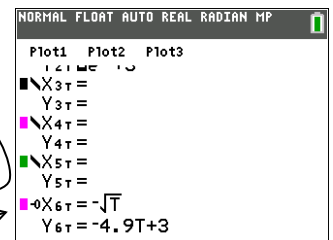
Then, during class, after displaying the graph of the Road Runner's and Wile E. Coyote's motion paths, sit your cursor on the = sign, press enter, and then press graph to have the TI-84 show Wile E. Coyote drop off the bridge into the river below.

Question 4 gives these parametric equations

$$x = -\sqrt{t}$$
$$y = -4.9t + 3$$

to students and ask them to eliminate the parameter by substitution.
In explicit form, we have $y = -4.9x^2 + 3$, $x \le 0$.
This is the position equation of a falling object.