Exploring the *n*th Roots of Unity with the Polynomial Root Finder App

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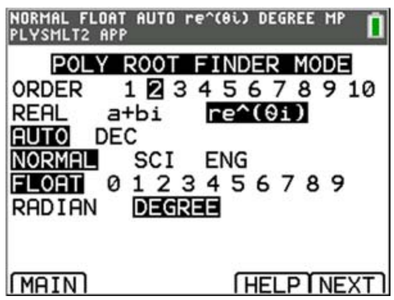
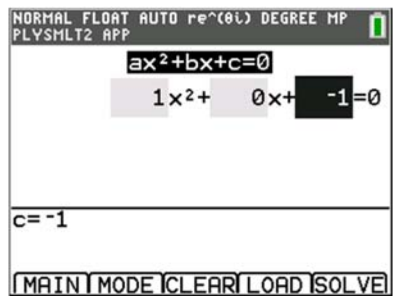
Using the PlySimul2 App we can explore the *n*th roots of unity.

Although we could type the coefficients into a template once we run the app (which we will do for $x^2 - 1$), a quick way to load the coefficients of the polynomial $a_n x^n - 1$ is by using lists, where L3 maps to $1x^3 + 0x^2 + 0x + -1 = x^3 - 1$, L4 maps to $1x^4 + 0x^3 + 0x^2 + 0x + -1 = x^4 - 1$, etc.

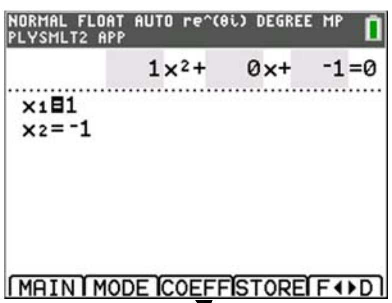
$x^3 - 1$	$x^4 - 1$	$x^5 - 1$	$x^6 - 1$
L3	L4	L5	L6
1	1	1	1
0	0	0	0
0	0	0	0
⁻1	0	0	0
------	⁻1	0	0
	------	⁻1	0
		------	⁻1

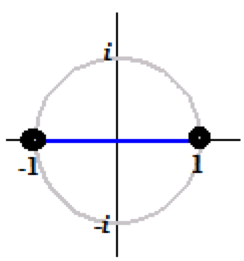
Use the settings below and press the [graph] key to select Next. Press [graph] again to select Solve.
In the second screen below (called the Mode Screen), the word DEGREE refers to measure of the angle θ when complex roots are displayed in the polar form $re^{\wedge}(\theta i)$.

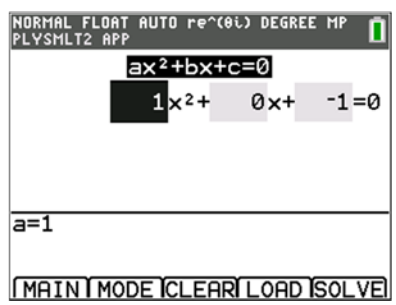




The solutions are displayed. Below is a plot of the two solutions on the complex plane.

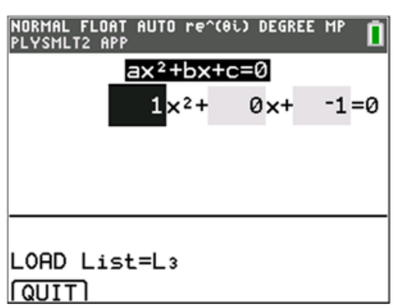


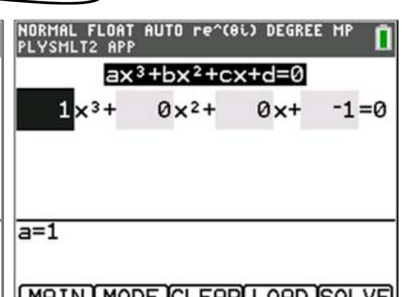




Since we have already created L3 ={1, 0, 0, –1}, we can press [zoom] for COEFF and then press [trace] for LOAD. (This will automatically set the ORDER of the polynomial for us. No need to do this if we are loading a list.)

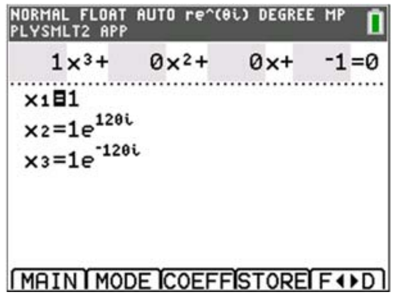
Type L3 (again, assuming it exists).

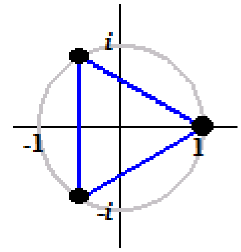




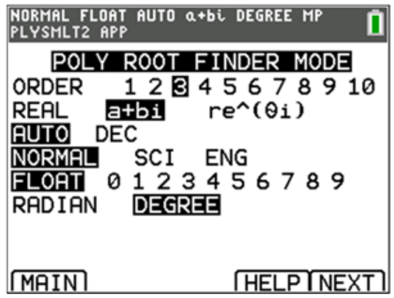
All the coefficients will load. Then press the [graph] key to select Solve.

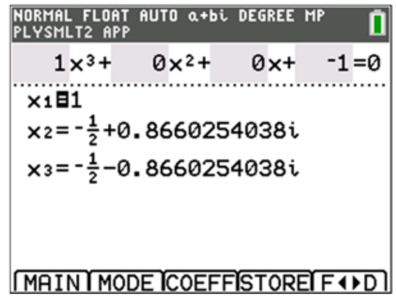
Per the settings in the app's Mode screen, the solutions are given in polar form in degrees. The cubed roots are vertices of the triangle inscribed in the unit circle.



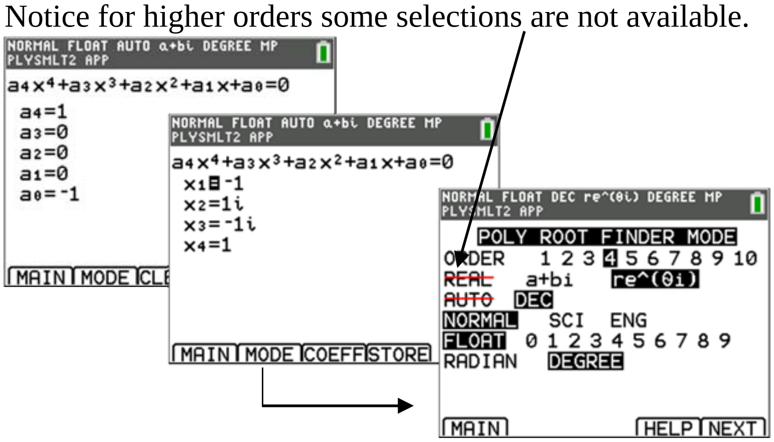


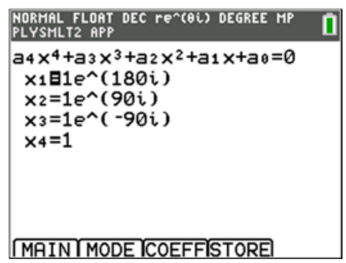
If desired, press [window] to select the MODE soft key and change to rectangular *a* +*bi*. Then press SOLVE to see the pair of conjugates.

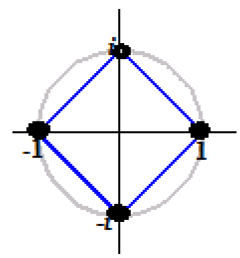


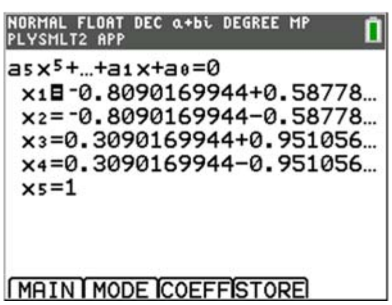


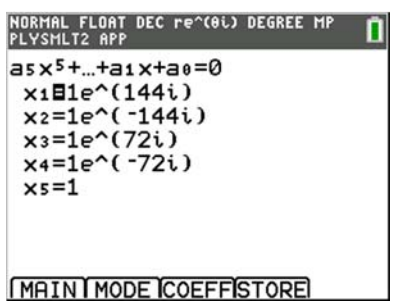
Repeat these steps for higher powers: Press COEFF, press LOAD, type the list, press SOLVE, change the mode to see the alternate form (polar and rectangular), press NEXT and then SOLVE.
Notice for higher orders some selections are not available.

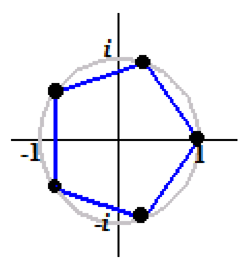


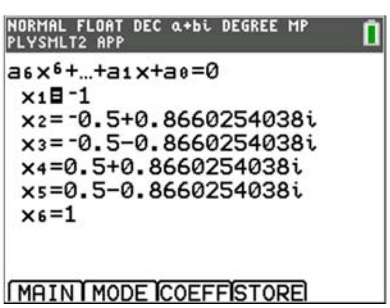


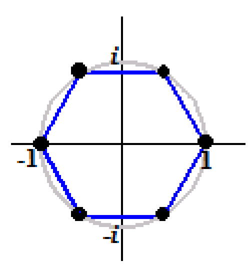








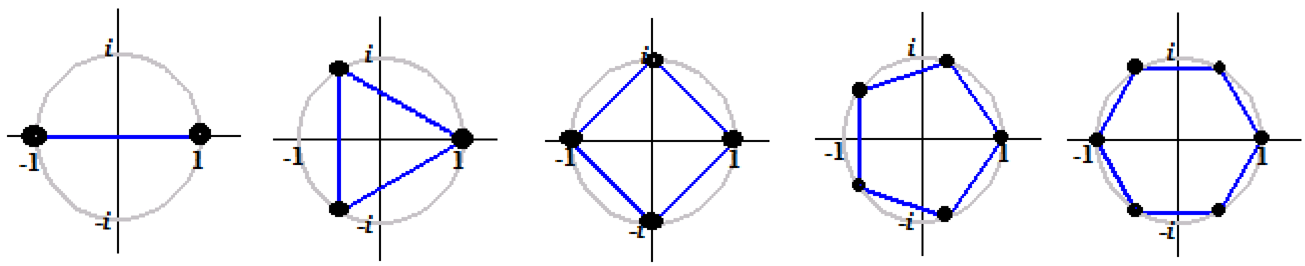




Possible Questions for Discussion

1. The number 1 is **always** a solution to $x^n - 1 = 0$. Why?

   However, the number –1 is **sometimes** a solution to $x^n - 1 = 0$. Under what conditions? Explain.



2. Examine the *n*th roots in rectangular form for **odd** values of *n*, i.e., $n = 3, 5, \ldots$
   Besides the solution $x = 1$, what do you notice about the pair(s) of the remaining solutions?
   Geometrically, what does this say about the location of these roots when plotted on the complex plane?
   What does this say about any symmetry?

3. For values of θ in degrees, the solutions to $x^n = 1$ are $x = 1e^{\theta i}$.
   Complete: when $n = 2$, θ = _____ and ______.
   when $n = 3$, θ = _____ , _____ , and ______.
   when $n = 4$, θ = _____ , _____ , _____ , and ______.
   when $n = 5$, θ = _____ , _____ , _____ , _____ , and ______.
   when $n = 6$, θ = _____ , _____ , _____ , _____ , _____ , and ______.
   In general, what is the measure of the arc, in degrees, that separates two neighboring *n*th roots on the unit circle for $n = 2, 3, 4, 5, \ldots$?