Taking Full Advantage of the Power of the TI-84 Plus

24th Annual T^3 International Conference
Chicago, Illinois

Friday, March 2, 2012
2:30 pm – 4:00 p.m.
San Francisco – West Tower – Gold Level

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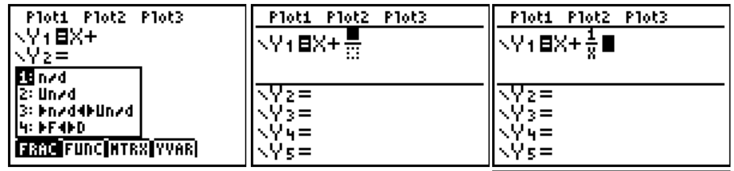
Look for and make use of structure. Look for and express regularity in repeated reasoning.

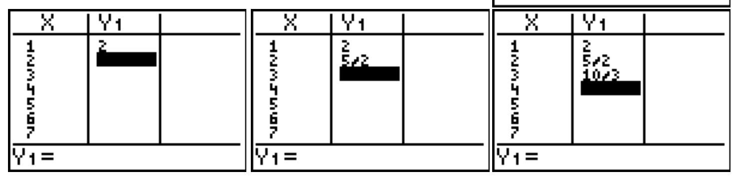
Investigation 1

a. Enter $x+\frac{1}{x}$ in Y1.
For the shortcut FRAC menu, press ALPHA [F1].



b. Set the table to start at 1, climb in steps of 1, automatically display the input, and display the output only when asked.

Sit your cursor over the first few outputs and press ENTER to display.

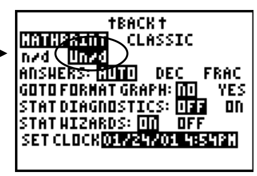


c. Using only the table of values, discuss the following

- What do you expect the next value to be?
- What pattern(s) do you see with the numerators? List as many patterns as you can find.
- Use the arrow keys and ENTER key to continue the table to see if your prediction is correct.

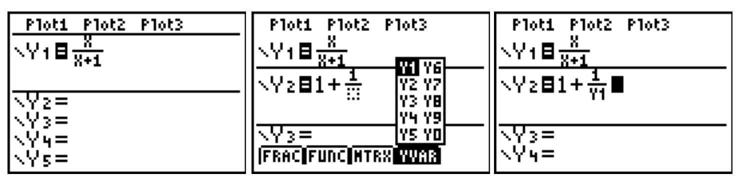
d. Use algebra to simplify the expression in Y1. What information does this simplified expression provide to help confirm or extend your observations in the previous question?

Change the mode to mixed **Un/d**.
Press ENTER on each output in the table. What connections do you see?

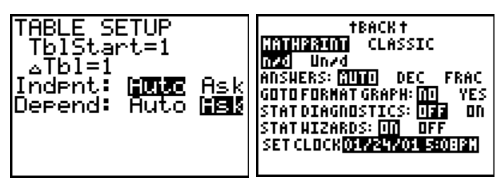


Investigation 2

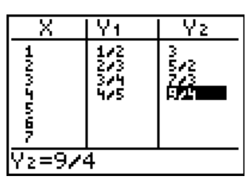
a. Enter $\frac{x}{x+1}$ in Y1. Enter $1+\frac{1}{\text{Y1}}$ in Y2.
For a shortcut to get Y1, press ALPHA [F4].



b. Keep the table settings as in the previous example.
Press Mode and select **n/d** to display improper fractions.



As before, sit your cursor over the first few outputs in Y1 and Y2 and press ENTER to display.



c. Repeat Investigation 1 c.

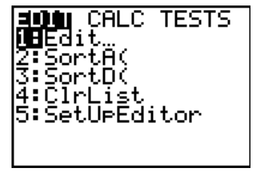
d. Repeat Investigation 1 d.

e. What connections do you see between the Investigation 1 and Investigation 2?

Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Model with mathematics. Use appropriate tools strategically.

<u>Investigation 3</u>

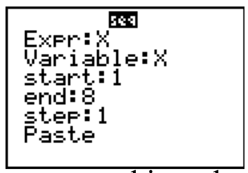
Congratulations! You are offered a job
where you are paid 1 measly dollar for the first day, but $2 for the second, $4 for the third, and so on,
so that each day's pay is double that of the previous day. How much total will you earn in eight days time?
How many days will it take for your total to exceed $100,000?



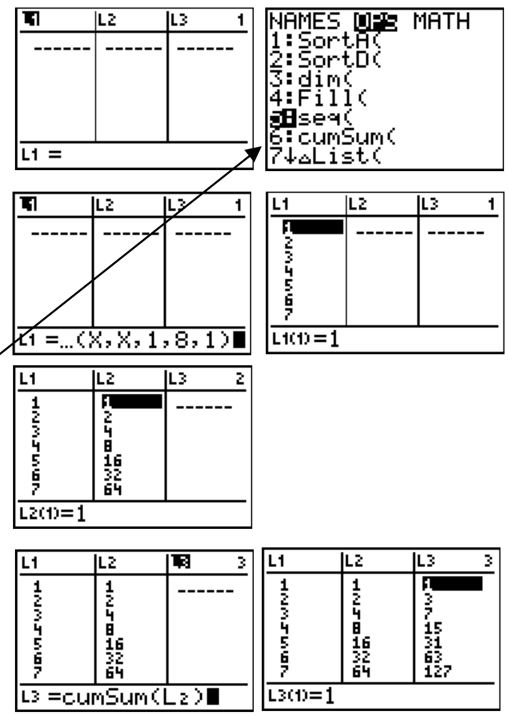
a. We can make a list which indexes the number of the day,
the amount you earned just that day, and a finally a cumulative sum.
Press STAT, followed by 1:Edit to get to the Stat Editor.

Tip: Set your cursor on L1 (the top shelf!) to make a sequence {1, 2, 3, …., 8} by pressing [2nd] [LIST] [▶] and 5: seq(.

The seq(wizard appears: Once the above settings are entered, highlight Paste and press [ENTER].



This builds and inserts the command into the L1 entry line.
Press [ENTER] once more to deliver the goods.



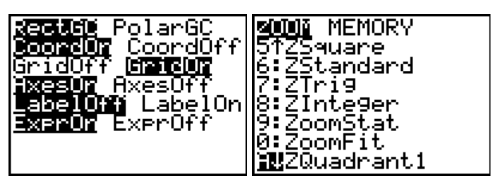
Build L2 to give the amount earned each day.

Build L3 which gives the cumulative sum.
Tip: Set your cursor on L3.
Press [2nd] [LIST] [▶] and 6: cumSum(
followed by [2nd] [L2] [ENTER].

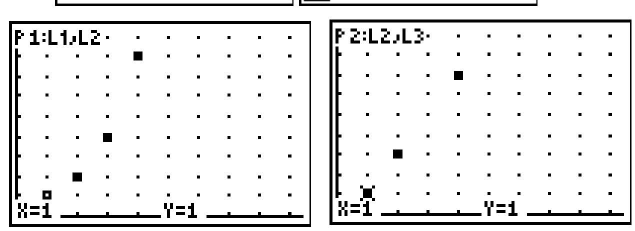
Scroll to see the amount on day 8.

b. Discuss what patterns you see in the table and in any plots. In particular:
   - Look for connections between L2 and L1. What kind of function would model (L1, L2)?
   - Look for connections between L3 and L2. What kind of function would model (L2, L3)?

c. TIP: To explore graphically,
press [2nd] [FORMAT] to display a dot grid.
Press [ZOOM] and scroll to use ZQuadrant1.



Construct formulas for the functions graphed below.



Construct a formula for the total earned (L3)
as a function of day # (L1).
Plot L3 vs. L1 and enter equation in Y1.
Then find the day that you first exceed
$100,000 in total earnings.

Spoiler alert if you flip the page!

Content Connections in Algebra and Precalculus for Investigation 3:

- Reasoning from the table (L1, L2), students may see that on day n the amount D earned that day would be $D = 2^{n-1}$. Along with a plot of data, other students may see that it can be modeled by the formula $D = a \cdot b^n$ with growth factor $b = 2$ and vertical intercept $a = \frac{1}{2}$, so $D = \frac{1}{2} \cdot 2^n$, which is equivalent to $D = 2^{n-1}$ by laws of exponents.

  The total (cumulative) sum S earned is a linear function of the amount earned that day, D, so $S = mD + b$. The average rate of change or slope $m = 2$ and the vertical intercept $b = -1$. Therefore $S = 2D - 1$.

  By substitution: $S = 2D - 1$
  $$= 2(\tfrac{1}{2} \cdot 2^n) - 1$$
  $$= 2^n - 1$$

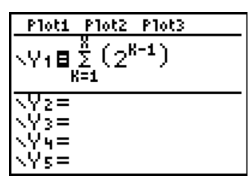
  The solution to $100,000 = 2^n - 1$ can be found through the table, graph, or analytically with logs.

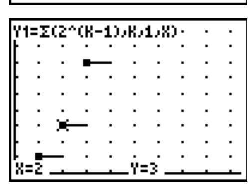
- On day 8 the total earned is the sum $S = 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2 + 1$. Students who have studied computers or base two arithmetic might recognize the base two representation of S as 11111111_2, which is a byte of 1's.

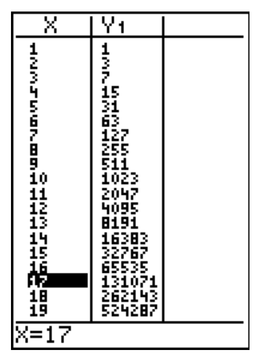
  If we add 1 to S, we have $S + 1 = 100000000_2 = 2^8 = 256$.
  So $S = 2^8 - 1 = 255$.

- Precalculus students who have studied geometric series and sigma notation could use another approach. On the nth day, the cumulative sum is $S = 1 + 2^1 + 2^2 + \ldots + 2^{n-1}$, a series of n terms.
  This could be written in sigma notation $S = 1 + 2^1 + 2^2 + \ldots + 2^{n-1} = \sum_{k=1}^{n} 2^{k-1}$

  We can enter this directly into Y1 and use a table solution.
  Observe what happens if you graph Y1.







  We could also derive its sum to show this formula is $S = 2^n - 1$:

$$S = \sum_{k=1}^{n} 2^{k-1} = 1 + 2^1 + 2^2 + \ldots + 2^{n-1}$$

Bye!

$$2S = \quad 2^1 + 2^2 + \ldots + 2^{n-1} + 2^n$$
$$-\quad S = 1 + 2^1 + 2^2 + \ldots + 2^{n-1}$$
$$2S - S = -1 \qquad + 2^n$$
$$= 2^n - 1$$

- It may surprise students to find when the sum will exceed 1 million, 1 billion, then 1 trillion, etc.
- Compare the above investigation with the question: *Find the sum of all of the positive divisors of 128.* $128 = 2^7$ and has factors 1, 2, 2^2, 2^3, 2^4, 2^5, 2^6, and 2^7. The sum is $S = 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2 + 1$.

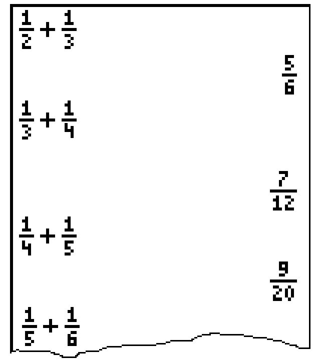
Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others.

Investigation 4

a. Use scrolling history and stacked fractions to produce the screen shown.

b. Continue the pattern. Will it work for $\frac{1}{20}+\frac{1}{21}$?

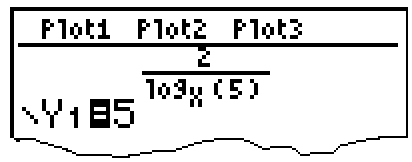
c. Discuss:
- What patterns do you notice?
- Will it always work? Justify with algebraic reasoning.



Investigation 5

a. Explore expressions of the form $a^{\frac{b}{\log_x a}}$

b. Insert the expression in Y1 with $a = 5$ and $b = 2$ and explore the table.

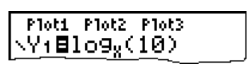


c. Explore and discuss:
- What happens when you change the parameter a to any positive number greater than 1?
- What happens when you change the parameter b to 3? to 1? to 0?
- Use properties of logarithms to explain.
  Hint: Take the logarithms to the base x of both sides of the equation $y = a^{\frac{b}{\log_x a}}$.

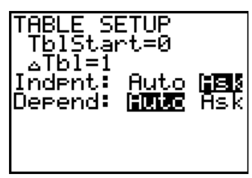
Thanks to T[3] Instructor John Hanna, http://www.johnhanna.us/ , who shared a similar problem which inspired this investigation.

Investigation 6

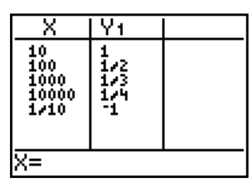
a. Consider the function $y = \log_x 10$.
Enter the expression in Y1.



b. Press 2nd WINDOW to match the screen shown to the right, where Indpnt is set to Ask.

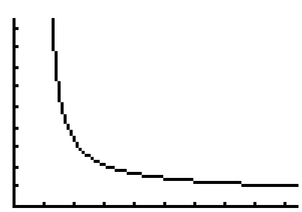


c. Explore with a table, where x is a power of 10.



Discuss:
- For the first four entries of the table, how is the denominator of the output related to the number of 0's of the input?
- What relationship holds for negative integer powers of 10, such as $\frac{1}{10}, \frac{1}{100}$, etc.?

d. Explore with a graph after, say, ZQuadrant1.



e. Rewrite the function $y = \log_x 10$ so that x is not the logarithmic base.
Hint: Let $y = \log_x 10$, write in exponential form, then take common logarithms of both sides of the equation. Compare tables and graph the result in the same window.

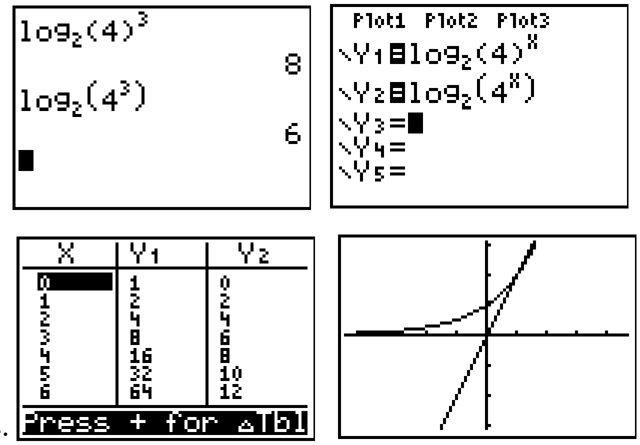
f. Follow up: In general, does $\log_a b = \frac{1}{\log_b a}$?

Hint: Let $y = \log_a b$, solve for b, then take logarithms of both sides to the base b.

Use appropriate tools strategically.

Investigation 7

a. Compare the expressions on the screen to the right.
Notice the usual order of operations are followed.
Unveil $\log_2(4)^3 = (\log_2(4))^3 = (\log_2 2^2)^3 = (2)^3 = 8$
and $\log_2(4^3) = \log_2(2^2)^3 = \log_2(2^6) = 6$



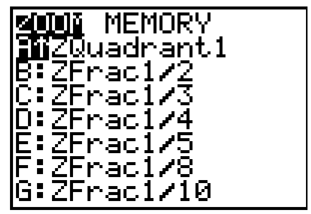
b. Explore with a table and a graph.
What is the simplified form of each?

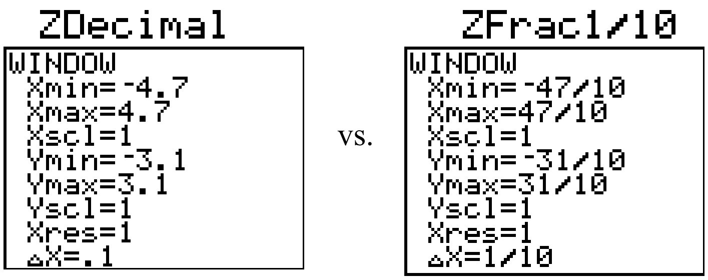
Do they look more familiar now?
Superimpose graphs of $y = 2^x$ and $y = 2x$ over each.

c. Facilitate a class discussion on logarithmic properties.

Investigation 8

Press ZOOM and scroll to see pre-defined windows with friendly pixel gaps.
Note: ZFrac1/10 sets the window variables so that you can trace in increments of $\frac{1}{10}$, if possible, and sets ΔX and ΔY to $\frac{1}{10}$.
Compare with ZDecimal which sets ΔX and ΔY to 0.1.

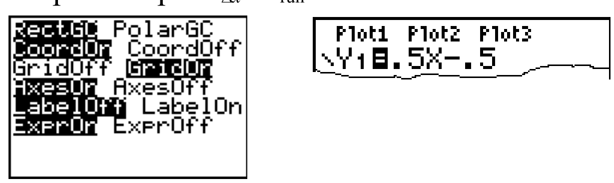




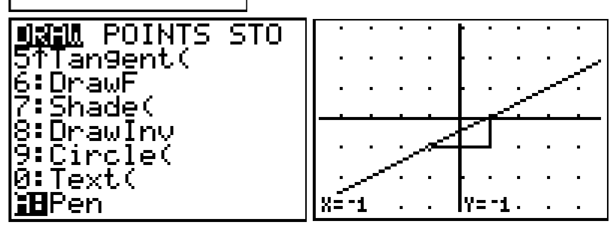
Provide an opportunity for hands-on exploration of the concept of slope $= \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$.

a. Press 2nd [Format] to select GridOn.

b. Graph $y = 0.5x - 0.5$.
Press ZOOM to select ZFrac1/10

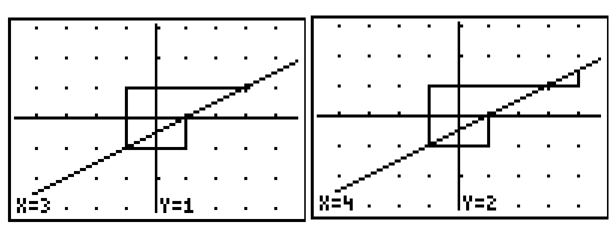


c. Press 2nd [Draw] to select Pen.
Press GRAPH to liberate the cursor from the line and observe the screen coordinates in this window.
Move to the point (1, 0), then press ENTER to start the pen.



d. Press the DOWN arrow key 10 times. You are at (1, −1). As you press, notice equivalent fractions.
Press the LEFT arrow key 10 times. You are at (0, −1).
Press the LEFT arrow key 10 more times. You are at (−1, −1) and back on the line.
Discuss the lengths of the legs of the "slope triangle."

e. Foster proportional reasoning by asking students to create additional slope triangles, showing
$$\frac{\text{rise}}{\text{run}} = \frac{-1}{-2} = \frac{2}{4} = \frac{1/2}{1} = \frac{1}{2}.$$

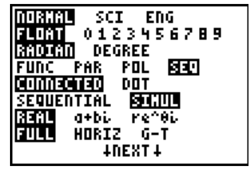


Look for and make use of structure. Look for and express regularity in repeated reasoning.

<u>Investigation 9</u>

Use the table in sequence mode to display the first few terms of a formula to the class. Students use recursive reasoning and problem solving strategies to find the next number in the sequence. For less difficult sequences, students also find the function defined explicitly and/or recursively.

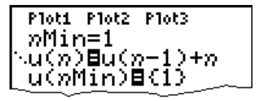
a. Do the following before class or without displaying to students:



i. Press MODE, highlight Sequence mode, then press [ENTER].

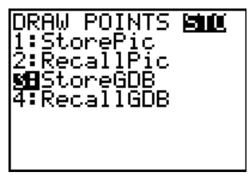
ii. Press [Y=]. Your graphing variable is now *n* instead of *x*.
You can get u, v, and w off the keypad. Enter the settings shown:

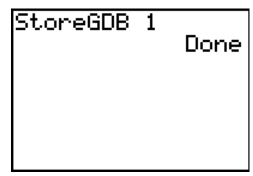
nMin = 1	*the beginning input*	
u(*n*) = *u*(*n*−*1*) + *n*	*the recursive rule* $u_n = u_{n-1} + n$	
u(*nMin*) = 1	*the beginning output* $u_1 = 1$	



iii. Press [2nd] [WINDOW] to match the screen shown to the right.

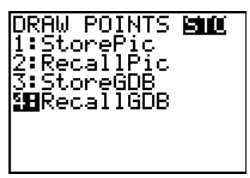
iv. To save as a graphical database (GDB) for later use in class, press [2nd] [DRAW] and arrow to the STO menu. Paste the command StoreGDB to the home screen, followed by 1 (for GDB #1) and then press [ENTER].

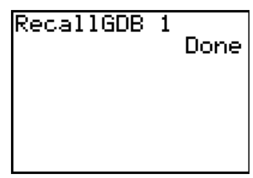




b. During class, divide students in pairs. Tell them you are going to display a pattern of numbers and you want them to find the next number in the sequence.

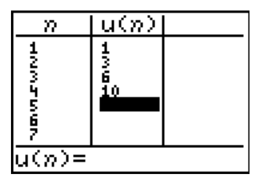
c. Recall the graphical database as shown.



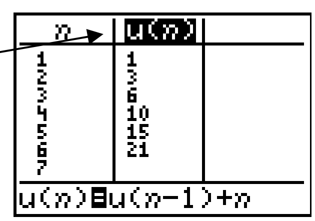


Show the table to the class, unveiling one output at a time, prompting students to guess the next one.

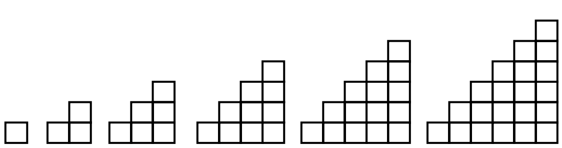
Urge them to think how the value could be found if we knew the one before it.



d. Once they have found the rule, you can show students the formula by walking the cursor up to the top of the second column.



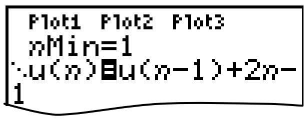
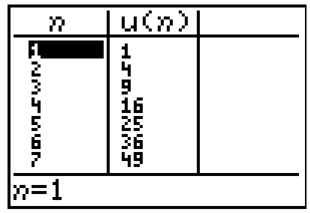
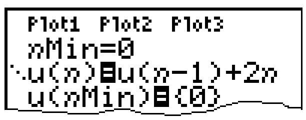
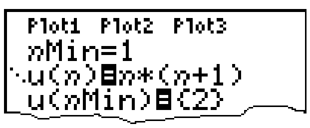
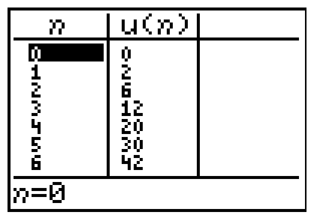
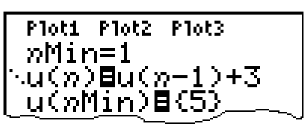
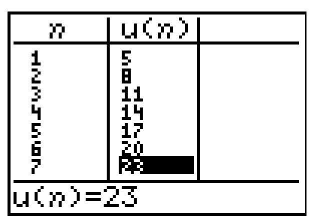
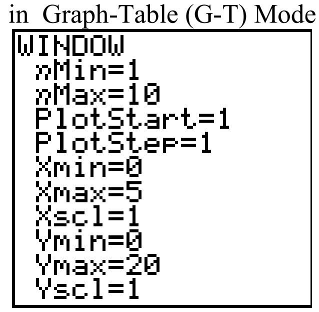
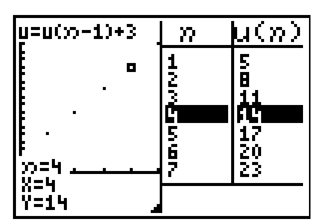
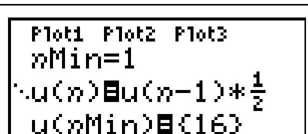
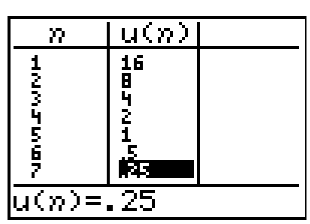
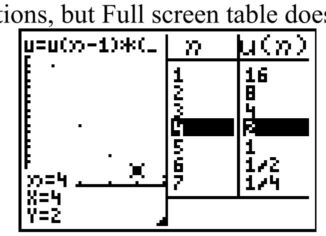
An equivalent rule is $u_n = n(n-1)/2$ with $u_1 = 1$.
Note: These are called the **triangular numbers**.



n:	1	2	3	4	5	6
u_n:	1	3	6	10	15	21

> The triangular numbers show up in several modeling scenarios.
> - *How many different handshakes are possible in a room with n people?*
> - *Seven people are entered in a table-tennis tournament. If each person plays one game with each of the other persons in the tournament, how many games will be played together?*

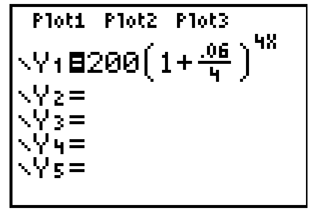
e. Give additional examples. Some ideas follow (or make up your own.) You can store each one in a different graphical database and recall it, or tinker with the rule while hiding the display from students.

Create this rule behind the scenes	Display this to students	Ask them to find what is next.
 $u_n = u_{n-1} + 2n - 1$ $u_1 = 1$		Most may recognize the **square** numbers, but if this example comes immediately after the triangular numbers, some students may notice the difference between successive values is 3, 5, 7, 9, respectively.
 $u_n = u_{n-1} + 2n$ $u_0 = 0$  $u_n = n(n+1)$ $u_1 = 2$		These are called the **rectangular** numbers or oblong numbers. Some may notice the difference between successive values is 2, 4, 6, 8, 10, respectively. Others may see the products: 0 x 1=0 1 x 2=2 2 x 3=6 3 x 4=12 Others may see they are twice the triangular numbers.
 $u_n = u_{n-1} + 3$ $u_1 = 5$		For Precalculus students, discuss that this is an arithmetic sequence with common difference 3 starting at 5, display a graph in Graph-Table (G-T) Mode  Press TRACE and left arrow. 
 $u_n = \frac{1}{2} u_{n-1}$ $u_1 = 16$		Notice G-T Mode shows thick bar fractions, but Full screen table does not. 

Use appropriate tools strategically.

<u>Investigation 10</u>

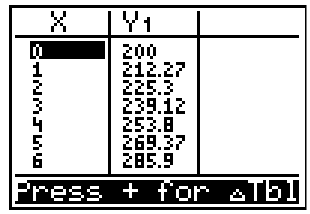
Find how long it takes for $200 compounded quarterly at 6 percent A.P.R. to grow to $475. Report your answer correct to the nearest 0.1 year.



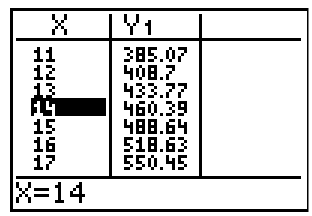
Use the ΔTable Shortcut.
Advantages: This is a quick way to find approximate solutions, since you often use the table to help build the graphing window anyway. It also provides an avenue for multiple perspectives.

a. Enter the expression in Y1 and press [2nd] [WINDOW] to match the screen shown to the right.

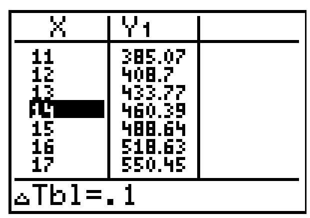
b. Scroll the table to find when the amount is closest to $475.

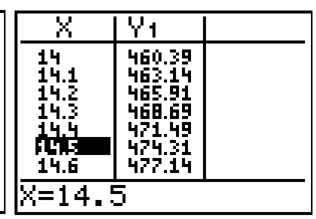


c. Position your cursor on the input whose output is closest to $475. In this case, we highlight 14.

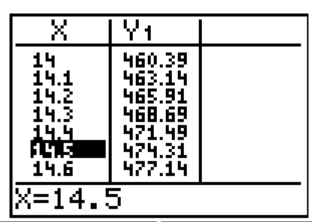


d. Press [+] and change ΔTbl to 0.1. Press [ENTER]. It will take about 14.5 years to reach $475.

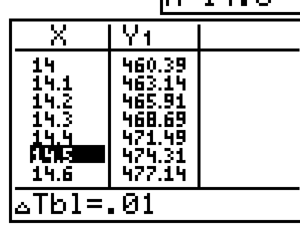


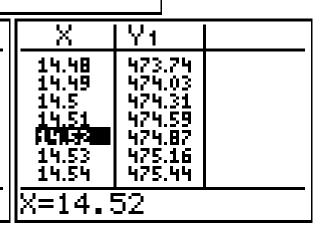


e. To approximate the answer to 0.01 years, we need only repeat the last two steps, setting ΔTbl to 0.01.



It will take about 14.52 years to reach $475. Support the answer with a graphical and analytical solution, or use the equation solver in the [MATH] menu.





Another example: Consider using the table to explore the behavior of $y = \dfrac{x^2 - 4}{x - 2}$ near $x = 2$.

Model with mathematics. Use appropriate tools strategically.

<u>Investigation 11</u>

a. Explore infinite geometric series.
For example, suppose a patient takes 4 mg of medication every day. After each dose, half of the medication in his body is metabolized.

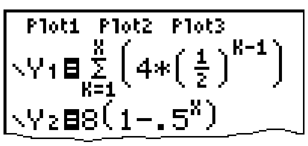
b. Discuss

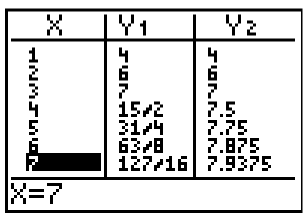
- What amount is in his body right after the second dose? The third? The *n*th?
- What happens in the long run?

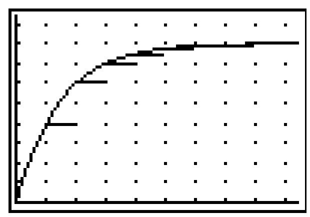
c. Students would find the amount after the first dose is $Q_1 = 4$.
After the second dose, the amount is $Q_2 = 4 + \frac{1}{2}(4) = 6$
After the third dose, the amount is $Q_3 = 4 + \frac{1}{2}(6) = 7$
Precalculus students might find the following for the amount in the body after the *n*th dose:

$$Q_n = 4 + 4(\tfrac{1}{2}) + 4(\tfrac{1}{2})^2 + 4(\tfrac{1}{2})^3 + \ldots + 4(\tfrac{1}{2})^{n-1} = \sum_{k=1}^{n} 4(\tfrac{1}{2})^{k-1} = \frac{4(1-\frac{1}{2}^n)}{1-\frac{1}{2}} = 8(1-\tfrac{1}{2}^n)$$

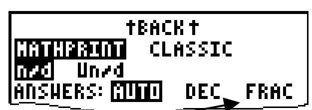
Graph in a ZQuadrant1 window with Dot Grid on to show stabilization occurs at 8 mg.
This can be confirmed with scrolling the table and examining the formula as $n \to \infty$.







Note: the answer display is in **Auto** Mode, which forces decimal output if it is present in the expression. Here a decimal output is more helpful in exploring the convergence to 8 mg.



Bad idea!

TIP: Setting the answer display to **Frac** could lead to misleading results in the same way that changing the mode from **Float** to **Fix 0** can give you misleading information.

The *Common Core State Standards for Mathematical Practice* describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.