Every MA 15400 course in Brightspace has the following lessons and learning objectives, accompanied by short videos, interactive figures, and online resources to practice.

Lesson: *The Graph of the Height of a Ferris Wheel Car with Time*

Overview

In this you will explore the graph of the height h of a Ferris Wheel Car as a function of time t, which is an example of a **periodic function**. We explore this graph and how it relates to the properties of the Ferris Wheel. This material is found in Section 7.1 of the course text.

Learning Objectives

1. Identify if a graph represents a periodic function.
2. Determine period, amplitude and midline.
3. Use a graph to find and interpret y if given t or vice versa

Lesson: *The Sine Function (and its Sidekick, Cosine)*

Overview

Every hero typically has a sidekick. Han Solo has Chewbacca. Sherlock Holmes has Dr. Watson. Our hero, the Sine Function, has the Cosine. In this we will explore how the sine and cosine function are defined in terms of the coordinates of a bug on circle centered at the origin (also called the unit circle). But first we will have a quick review of angles and their properties. This material is in Section 7.2 in the text.

Learning Objectives

1. Sketch the position of a point on a circle of radius r corresponding to a given angle (or value of time or number of revolutions) and give its coordinates
2. Find the (x, y) coordinates of a point on the circle if given its angle.
3. Find angles between 0° and 360° which have the same sine or cosine of a given angle.
4. Determine in which quadrant an angle lies if given certain conditions.

Lesson: *Radians*

Overview

Have you ever wondered why a full revolution around a circle had been decreed to be 360° instead of, say any other convenient number? If you are curious, this Web page gives a possible answer. We are now so accustomed to degrees that we may be surprised that there is another way to measure angles, namely in **radians**. In fact, we will see that working in radians will turn out to be very convenient! In the definition of a radian, the **RADI**us of the circle is involved to make the measurement of the **AN**gle, which is where the **RADIAN** gets its name. This material is in Section 7.3 in the text.

Learning Objectives

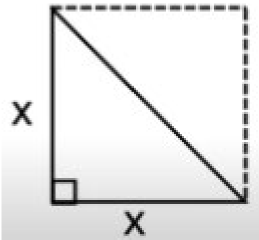
1. Interpret the radian measure of the central angle of a circle of radius r as the number of radius lengths, r, that you need to wrap around the rim of the circle on the arc spanned by the angle.
2. Find the location of an angle θ that is a fraction of π, such as $\pm\pi/6$, $\pm\pi/4$, $\pm\pi/3$, $\pm\pi/2$, or multiples of these.
3. Convert an angle from degrees to radians and vice versa.
4. If given two of the arc length s, radius r, or an angle θ, find the third by using the relationship between them.

Lesson: *Special Right Triangles*

Overview

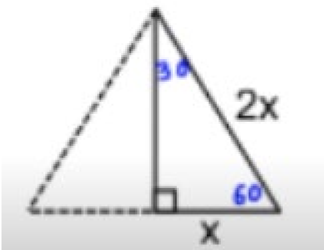
Squares and equilateral triangles are very popular in the field of engineering and more advanced mathematics.

- If a square is cut in half along its diagonal we will create two isosceles right 45°- 45°-90° right triangle.



- If an equilateral triangle is cut in half from a vertex to its opposite side we will create two 30°- 60°-90° right triangle.

Notice an important property: the hypotenuse is double the length of the short leg.



Proportional reasoning is simply this: we multiply the same number to each side of the above special triangles to find all three sides if given only one of them. We call the 30°-60°-90° and the 45°-45°-90° right triangles the "special right triangles". This material is in Chapter 7 Skills Refresher in the text.

Learning Objectives

1. Use proportional reasoning to find the exact values of the 3 sides of an isosceles right 45°- 45°-90° triangle if given one of the sides.
2. Use proportional reasoning to find the exact values of the 3 sides of a 30°- 60°-90° right triangle if given one of the sides.

Lesson: *Trigonometric Values of Special Angles*

Overview

The "special right triangles" are the 30°-60°-90° and the 45°-45°-90° triangles.
The "special angles" are multiples of 30°, 45°, and 60° (or $\pi/6$, $\pi/4$, or $\pi/3$, respectively).
In the Lesson on *Radians* we were able to find the value of sin θ and cos θ if θ was a quadrantal, i.e., θ is a multiple of 0°, 90°, 180°, 270°, and 360° (or 0, $\pi/2$, π, $3\pi/2$ or 2π) which corresponds to the 3 o'clock, 12 o'clock, 9 o'clock, or 6 o'clock position. From the previous lesson, we found the value of the legs of the special triangles with hypotenuse 1. This enables us to find the **exact** coordinates of the bug's location for the special angles without a calculator. (The exact values are non-decimal and, in this case, the square root symbol may be involved.) Recall that the x-coordinate of the bug's location is the cosine of the angle and the y-coordinate of the bug's location is the sine of the angle.

Learning Objectives

1. Find exact values of sine and cosine for multiples of 30°, 45°, and 60° without a calculator.
2. Find exact values of sine and cosine for multiples of $\pi/6$, $\pi/4$, or $\pi/3$ without a calculator
3. Given a sketch of an angle θ that is a multiple of 30°, 45°, and 60° or $\pi/6$, $\pi/4$, or $\pi/3$, report the value of θ as well as its sine and cosine.

Lesson: *Graphs of Sine and Cosine and Outside Changes to the Function*

Overview

In the first week of this course we sketched the graph of the height of the Ferris Wheel car as a function of time, starting at the 3 o'clock position. In a similar way, we plot the *y*-coordinate of the bug on the unit circle as function of the angle to produce the graph of the sine function. We then produce a graph of the cosine function by plotting the *x*-coordinate of the bug as a function of the angle. We see both functions share similar properties. Finally, we look at "outside" changes to the function. This material is in Section 7.4 in the text.

Learning Objectives

1. Report the main characteristics (period, amplitude, midline, domain, range, odd/even symmetry, when it is positive, negative, increasing, decreasing, if it starts at or above the midline) of the graph of $y = \sin\theta$, and $y = \cos\theta$.
2. Relate the graph of the sine or cosine function to the unit circle as the *x*-coordinate (cosine) or the *y*-coordinate (sine) of the point on the circle.
3. For $y = A\sin(x) + k$ or $y = A\cos(x) + k$, identify the period, amplitude, and midline.

Lesson: *Inside multiplicative change*

Overview

Previously, we looked at "*out*side" changes to the function y = sin(x), namely $y = A\sin(x) + k$, which affected the *out*put.In this we look at "*in*side" changes, which will affect the *in*put.
We will first consider a *multiplicative inside* change to $y = \sin(x)$ or $y = \cos(x)$ by exploring the effect of B on the function $y = \sin(Bx)$ or $y = \cos(Bx)$. In the next we will explore an *additive inside* change to the function. This material is in Section 7.5 in the text.

Learning Objectives

1. Report the period of the graph of $y = A\sin Bx + k$.
2. If you have found the period, p, of $y = \sin Bx$, check that your value is correct by substituting into the formula and verifying that $B\cdot p = 2\pi$.
3. Given a graph, report the midline, amplitude and period and use them to find the formula $y = A\sin Bx + k$ or $y = A\cos Bx + k$.

Lesson: *Review of horizontal shifts to a function (optional)*

Overview

The horizontal shift to the left or to the right of the graph of a parent function affects the formula of the child function, but in a way that sometimes students may not expect. On the bright side, if you forget whether to use a plus or a minus sign, you can always check your answer with a grapher and revise if needed.

Learning Objectives

1. Write the formula of function $y = f(x)$ which has been shifted h units to the right as $y = f(x - h)$.
2. Write the formula of function $y = f(x)$ which has been shifted h units to the left as $y = f(x + h)$.

Lesson: *Inside Additive Change - Phase Shift*

Overview

The phase shift is a very handy way to describe how a sinusoidal graph (regular or upside down sine or cosine) has moved to the left or to the right. In this we will see how to find it from a graph using a grid, and then how it is incorporated into the formula of the function. Note: the phase shift can be found whether or not we know the period or any values on the horizontal axis of the graph. It is a relative shift to the period. This material is in Section 7.5 in the text.

Learning Objectives

1. Given a graph and a model choice (regular or upside down sine or cosine) report the phase shift.
2. Given a graph with a nonzero phase shift φ, find a formula $y = A\sin(Bx-\varphi)$ or $y = A\cos(Bx-\varphi)$.

Lesson: *Inside Additive Change - Horizontal Shift*

Overview

In addition to the phase shift, we can use the horizontal shift to describe how a sinusoidal graph (regular or upside down sine or cosine) has moved to the left or to the right. We will see how the horizontal shift h is incorporated into the formula of the function as $y = \sin(B(x-h))$ and is related to the phase shift φ, where $B\cdot h = \varphi$ to create a formula $y = \sin(B(x-h)) = \sin(Bx-\varphi)$. Notice where the parentheses are placed is important. This material is in Section 7.5 in the text.

Learning Objectives

1. Sketch the graph of a function if given its formula $y = A\sin(B(x-h)) + k$ or $y = A\sin(Bx-\varphi) + k$.
2. Given a graph with a nonzero phase shift φ or horizontal shift h, find a formula $y = A\sin(B(x-h)) + k$ or $y = A\sin(Bx-\varphi) + k$.
3. If you have found the phase shift, φ, of the graph of $y = \sin(Bx-\varphi)$, check that your value is correct by multiplying out $y = \sin(B(x-h))$ and verifying that $B\cdot h = \varphi$, where h is the horizontal shift.

Lesson: *The Tangent Function*

Overview

Just as we plotted the y-coordinate of the bug on the unit circle as function of the angle to produce the graph of the sine function and then plotted the x-coordinate of the bug as a function of the angle to produce the graph of the cosine function, we will plot the slope y/x of the terminal side of the angle to produce the tangent function and explore its properties. Finally, we look at multiplicative changes to the function. This material is in Section 7.6 in the text.

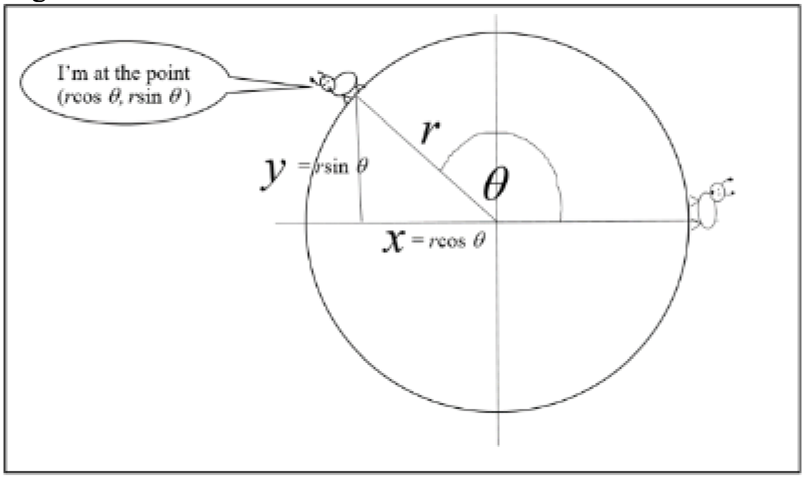
Learning Objectives

1. Report the main characteristics (period, domain, range, symmetry, value at $\pi/4$, when it is positive, negative, zero, undefined. increasing) of the graph of $y = \tan\theta$.
2. Relate the graph of $y = \tan\theta$ as the slope y/x of the terminal side of θ that corresponds to the point (x, y) on the unit circle.
3. Given the graph of $y = A\tan Bx$ and its intercepts and vertical asymptotes, find A and B.
4. Given the formula of $y = A\tan Bx$, report its intercepts and vertical asymptotes (exact). Solve $A\tan Bx = A$.

Lesson: *The Reciprocal Functions*

Overview

Recall the sagacious words of the bug on the circle of radius r after a trip on the rim corresponding to an angle θ shown below.



(If desired, click on the image to enlarge it.)

Since $x = r\cos\theta$ and $y = r\sin\theta$ then, in general for θ in any quadrant, we have the following: $\cos\theta = x/r$ and $\sin\theta = y/r$. We also have the slope of the terminal side of θ given by $\tan\theta = r\sin\theta / r\cos\theta = \sin\theta / \cos\theta = y/x$ for $x \neq 0$. These ratios will be used to define and find the reciprocal functions. This material is covered in Section 7.7 in the text.

Learning Objectives

1. Find exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, $\cot\theta$ if given the angle θ as a multiple of 30°, 45°, and 60° (or $\pi/6$, $\pi/4$, or $\pi/3$).
2. Find exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, $\cot\theta$ if given the value of one of these trig functions using the Pythagorean Theorem.
3. If given the location of θ use the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$ to find the value of $\sin\theta$ (and its sign) if given $\cos\theta$ or vice versa.

Lesson: *Inverse Trig Functions*

Overview

When we see the notation $\sin^{-1}(w)$ our eyes can be deceived. While it may *look* like the reciprocal of the sine of w, it means actually something quite different. In fact, $\sin^{-1}(w)$ is equivalent to the angle whose sine is w. We actually call this the inverse sine of *w* or the arcsine of *w*. The inverse trigonometric function inverse sine, inverse cosine, and inverse tangent, are also called *arc*sine, *arc*cosine, and *arc*tangent. Note *arc* is synonymous for **angle**. The inverse functions always output an **angle**. In this series of videos we define these functions and investigate their graphs. This is covered in Section 7.8.

Learning Objectives

1. Solve simple trig equations over a requested interval; for example $[0, 2\pi)$ or [0,360°) or other intervals, providing **a.** exact values of angles measured in radians (when given special angles which are multiples of 30°, 45°, 60° or their radian equivalents.) **b.** decimal approximations using the inverse trig functions.
2. Distinguish the meaning of the notation $\sin^{-1}x$, $\arcsin x$, $\sin^2 x$, $\sin x^2$, $\csc x$, etc.

Lesson: *Right Triangle Trigonometry*

Overview

We have seen the definitions of $x = r\cos\theta$, $y = r\sin\theta$, and $\tan\theta = y/x$ = RISE/RUN = slope from the bug on the circle.We show how they are the same as the right triangle definitions which involve the hypotenuse and the two legs, which are named by their location to the angle θ, i.e., opposite and adjacent. As always, Pythagoras lurks in the background. This material is in Section 8.1 in the text.

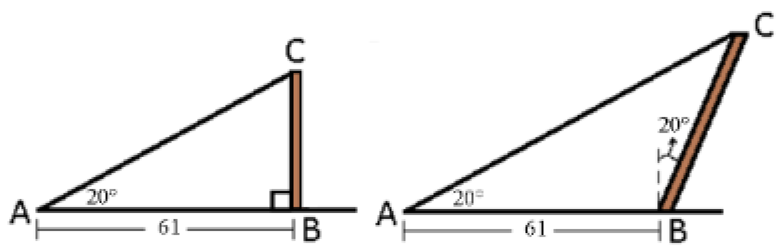
Learning Objectives

1. If given any two of the side lengths of a right triangle, find the remaining parts.
2. If given a side length and an angle of a right triangle, find the remaining parts.
3. Solve application problems which involve right triangles.
4. Interpret $\tan\theta$ as the slope of the angle of inclination.

Lesson: *The Law of Sines*

Overview

If you have perfectly level ground and a perfectly vertical tree, you can use the right triangle definitions to find its height if you know an angle and the distance away. However, what happens if your tree leans to an angle or your ground is not level? Another tool will be needed and so we call upon the Law of Sines. This material is in Section 8.2 in the text.



Learning Objectives

1. Solve for sides and angles of a triangle using the Law of Sines.
2. Determine when you can use right triangle trigonometry (SOHCAHTOA) paired with Pythagoras and when you can use the Law of Sines.
3. Solve application problems which involve the Law of Sines.

Lesson: *The Law of Cosines*

Overview

Poor cosine. It doesn't even have it's own name (Co-sine = "I'm With Sine"). After Sine was given its own Law, which we showed last lesson, the Cosine wanted it own law too. So, to appease Cosine, we have this section. However, as you will see in the videos, this Law is really just the Pythagorean Theorem extended to non-right triangles. And given the image of Bruce the shark above, this could also be called the "Law of Jaws". This material is in Section 8.2 in the text.

Learning Objectives

1. Solve for sides and angles of a triangle using the Law of Cosines.
2. Determine when you can use the Law of Cosines and when you can use the Law of Sines.
3. Solve application problems which involve the Law of Cosines

Lesson: The Ambiguous Case of the Law of Sines

Overview

It is possible for two angles to have the same sine in the interval $0° \leq \theta \leq 180°$. For that reason, it may be possible for two triangles to exist when finding a missing angle with the Law of Sines. The word "ambiguous" is used to indicate that there could be 1, 2, or no triangles if given SSA (side-side-angle). This material is in Section 8.2 in the text

Learning Objectives

1. Solve problem situations involving the ambiguous case of the Law of Sines.
2. If given a situation which requires finding an angle with the Law of Sines, determine if one, two, or no triangles exist.

Lesson: *Solving Trig Equations Graphically*

Overview

The advantage of of being able to solve an equation graphically is that we can see the number of solutions that exist over a specified interval. Because of symmetry, we can use the periodicity of the sine or cosine function to find all possible solutions. This material is in Section 9.1 in the text.

Learning Objectives

1. Solve a trig equation which involves sine or cosine graphically.
2. Use symmetry and the period of the function to find all solutions to a trigonometric equation.

Lesson: *Trig Identities*

Overview

In this chapter we introduce *Analytical Trigonometry* which is a marriage of the laws of algebra (mostly involving fractions) with the concepts of trigonometry. In this section in particular we see how to create identical trigonometric expressions, which are also called trig identities. This material is in Section 9.2.

Learning Objectives

1. Rewrite trigonometric expressions.
2. Build fractional fluency.

Lesson: *The Pythagorean Identities and the Double Angle Identities*

Overview

Because of the connection between the unit circle, the Pythagorean Theorem, and the identity $\cos^2\theta + \sin^2\theta = 1$, the latter is called a Pythagorean Identity. This identity is so prodigious that it spawns the additional Pythagorean Identities shown in this lesson. We also share the double angle identities $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$ (for now, without proof) and how Pythagoras can provide variations of the double angle formula for cosine. This material is in Section 9.2 in the text.

Learning Objectives

1. Use the double angle trig identities for $\sin 2\theta$ and $\cos 2\theta$ to rewrite trig expressions.
2. Use the Pythagorean Identities $\cos^2\theta + \sin^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $\cot^2\theta + 1 = \csc^2\theta$.
3. Write trigonometric expressions into algebraic expressions.
4. Continue to build fractional fluency.

Lesson: *Solving Trig Equations*

Overview

One of the uses of trig identities is that they can be helpful in solving trig equations algebraically. We solve trig equations by factoring as well as by using a graph. Some of these equations may or may not need the substitution of an identity. This material is in Section 9.2 in the text.

Learning Objectives

1. Use the trig identities to solve trig equations algebraically using the unit circle.
2. Use a grapher to determine the solutions to a trig equation.

Lesson: *Using Sum and Difference Identities*

Overview

The sum and difference identities enable us to show why the double angle identities are true. They also can help us simplify trigonometric expressions. This material is in Section 9.3 in the text.

Learning Objectives

1. Recognize the sum and difference identity for sine and use it to simplify expressions.
2. Recognize the sum and difference identity for cosine and use it to simplify expressions.
3. Recognize the sum and difference identity for tangent and use it to simplify expressions.

Lesson: *Polar Coordinates*

Overview

When we specify our location, we often use coordinates. In the past, rectangular or Cartesian coordinates have been our custom, but if we could fly we could specify our location with polar coordinates. This concept is a review of earlier material, even though it is presented as if it is a new topic. This material is in Section 8.3 in the text.

Learning Objectives

1. Convert coordinates from polar to rectangular.
2. Convert coordinates from rectangular to polar.

Lesson: *Function Composition and Decomposition*

Overview

We can compose one function with another, which is just a fancy name for **substitution**. We can also reverse the process, which we call function decomposition. This material is in Section 10.1 in the text.

Learning Objectives

1. If given the formulas for $f(x)$ and $g(x)$, find the formula for $f(g(x))$, i.e., f composed with g.
2. If given the formula for $h(x) = f(g(x))$, find possible formulas for $f(x)$ and $g(x)$, i.e., decompose h.
3. If given the formula for $h(x) = f(g(x))$ and $f(x)$, find the formula for $g(x)$.
4. If given the formula for $h(x) = f(g(x))$ and $g(x)$, find the formula for $f(x)$.

Lesson: *Inverse Functions*

Overview

We review the meaning and notation of inverses, one-to-one functions, and finding the formula of an inverse function. This material is in Section 10.2 in the text.

Learning Objectives

1. Determine if a function is invertible.
2. Find the formula for the inverse function.

Lesson: *Graphical Representation of Vectors*

Overview

If you have ever used a people mover in an airport (or moving sidewalk), then you have performed *vector addition*. We will explore this concept in the set of videos below. This material is in Section 12.1 in the text.

Learning Objectives

1. Given the sketch of a vector, use a grid to sketch the scalar multiplication or the opposite of the vector.
2. Given the sketch of vectors, use a grid and the "head to tail" method to sketch the result of vector addition or subtraction.

Lesson: *Component Form of Vectors*

Overview

In the previous we used a triangular grid. In this we use a rectangular grid to add and subtract vectors using components. This material is in Section 12.1 and 12.2 in the text.

Learning Objectives

1. Resolve a vector into horizontal and vertical components given its magnitude and direction.
2. Given the horizontal and vertical components of a vector, report its magnitude and direction.
3. Perform vector arithmetic.

Lesson: *Applications of Vectors*

Overview

We have already seen some applications of two dimensional vectors. We will show an example of three dimensional vectors as well as an application involving combining two forces. This material is in Section 12.3 in the text.

Learning Objectives

1. Perform vector arithmetic for a vector of n-dimensions, $n > 2$.
2. Use vectors to combine forces.

Lesson: *Introduction to Sequences*

Overview

We provide both an informal and a formal definition of a sequence. We share examples of several popular sequences, two of which are so popular they are given special names - *arith**met**ic* and *geo**met**ric*. (Both are pronounced with the stress on the third syllable.) This material is in Section 13.1 in the text.

Objective

1. If given some terms of a sequence, find the next term.
2. Classify a sequence as arithmetic, geometric, or neither.

Lesson: *Finding the Formula for the nth Term of a Sequence*

Overview

We provide both an informal and a formal definition of a sequence. We share examples of several popular sequences, two of which are so popular they are given special names - *arith**met**ic* and *geo**met**ric*. (Both are pronounced with the stress on the third syllable.) This material is in Section 13.1 in the text.

Learning Objectives

1. If given some terms of a sequence, find the next term.
2. Classify a sequence as arithmetic, geometric, or neither.
3. If given a plot of the terms (n, a_n) of an arithmetic sequence, find the equation of the linear function $y = mx + b$ which passes through the points.
   Relate the slope m to the common difference $d = a_2 - a_1$ of the sequence. Relate b to what would be* the term a_0 prior to the first term, i.e., $a_0 + d = a_1$.
4. If given a plot of the terms (n, a_n) of a geometric sequence, find the equation of the exponential function $y = ab^x$ which passes through the points. Relate the growth factor b to the common ratio $r = \frac{a_2}{a_1}$ of the sequence. Relate a to what would be the term a_0 prior to the first term, i.e., $a_0 \cdot r = a_1$.

Lesson: *Introduction to Series and Sigma Notation*

Overview

When we replace the commas of a sequence with plus signs we have what is called a series. The sum of the terms of an *arithmetic sequence* is called an *arithmetic series*. The sum of the terms of a *geometric sequence* is called a *geometric series*. We show a way to write series using sigma notation. This material is in Section 13.2 in the text.

Learning Objectives

1. Find the sum of an arithmetic series using an efficient strategy (like what Gauss did).
2. If given a series in expanded notation, write it in sigma notation or vice versa.

Lesson: *The Sum of a Finite Geometric Series*

Overview

In the previous section we use a technique similar to what Karl Gauss likely used to find the sum of a finite arithmetic series. In this section we use a show a formula to find the sum of a finite geometric series. This material is in Section 13.3 in the text.

Learning Objectives

1. Determine the sum of a finite geometric series using the **formula**.
2. Solve application problems involving finite geometric series.

Lesson: *The Sum of an Infinite Geometric Series (for 0 < r < 1)*

Overview

In this section we look at geometric series with an infinite number of terms. If the ratio r of consecutive terms is less than 1, the sum will actually approach a value. This material is in Section 13.4 in the text.

Learning Objectives

1. Determine the sum of a infinite geometric series using the **formula**.
2. Solve application problems involving infinite geometric series.

Lesson: *Parametric Equations*

Overview

Parametric Equations enable us to construct graphs which model the path of an object, for example, the robot that landed on Mars. This material is in Section 14.1 in the text.

Learning Objectives

1. Given graphs of two parametric equations $x = f(t)$ and $y = g(t)$, identify the path of the object (y vs. x).
2. Given equations of two parametric equations $x = f(t)$ and $y = g(t)$, eliminate the parameter to write y in terms of x.

Lesson: *How to Use a Grapher to Sketch Parametric Equations*

Overview

We show how to set your grapher in parametric mode and what you need to know to create a graph. This material is in Section 14.2 in the text.

Objective

1. Change the mode on a grapher from function mode to parametric mode.
2. Use the window settings on a grapher to produce a sketch of a set of parametric equations.

Lesson: *Circles*

Overview

We have graphed $f(t) = A\cos(t) + H$ and $g(t) = A\sin(t) + K$ **in a previous lesson many weeks ago**, where $|A|$ is the amplitude. In this we connect $|A|$ to the radius r of a circle which is the path of ($f(t)$, $g(t)$). We connect the midline (which is middle of the range of $f(t)$ and $g(t)$) to the center (H, K) of the circle. Symmetry plays a role. This material is in Section 14.2 in the text.

Learning Objectives

1. Given the implicitly defined formula of a circle, report its center and radius
2. Given the graph of a circle, report its center, radius, and formula in implicit and parametric form.

Lesson: *Ellipses*

Overview

For $x = f(t) = A\cos(t) + H$ and $y = g(t) = A\sin(t) + K$, the amplitude $|A|$ is the radius r of a circle and the midlines of each function (which is middle of the range of $f(t)$ and $g(t)$) give the center (H, K) of the circle. We can do this by utilizing the Pythagorean Identity $\cos^2 t + \sin^2 t = 1$. To parametrize an ellipse, we just make the amplitude of $f(t)$ *different* than the amplitude of $g(t)$. One could call the the amplitude of $f(t)$ the "horizontal radius" and the amplitude of $g(t)$ the "vertical radius". However, for purposes which will make more sense in the next section with hyperbolas, we will call them the **RUN** and **RISE**, respectively. Instead of a "larger diameter" and a "smaller diameter," we will use the words **major axis** and **minor axis**, respectively. The two endpoints of the major axis are called the **vertices**. This material is in Section 14.3 in the text.

Learning Objectives

1. Given the implicitly defined formula of a ellipse, report its center, vertices, domain, range, RUN, RISE, and length of the major and minor axes. Sketch its graph.
2. Given the graph of an ellipse, report its center, vertices, RUN, RISE, and length of the major and minor axes.
3. Given the graph of an ellipse, write a formula in implicit and parametric form.

Lesson: *Hyperbolas Compared and Contrasted with Ellipses*

Overview

We have seen that the parametric equations $x = f(t) = RUN\cos(t) + H$ and $y = g(t) = RISE\sin(t) + K$ can be written implicitly as $\frac{(x-h)^2}{RUN^2} + \frac{(y-k)^2}{RISE^2} = 1$. We can do this by utilizing the Pythagorean Identity $\cos^2 t + \sin^2 t = 1$. If we change the + sign in the implicit formula to $-$, we will have a hyperbola. Since it makes a difference in order when subtracting (for example, 7-5 is different than 5 - 7), we have two different possibilities. Namely, we have $\frac{(x-H)^2}{RUN^2} - \frac{(y-K)^2}{RISE^2} = 1$ and $\frac{(y-K)^2}{RISE^2} - \frac{(x-H)^2}{RUN^2} = 1$. This material is in Section 14.4 in the text.

Learning Objectives

1. Given the implicitly defined formula of a hyperbola, report its center, vertices, RUN, and RISE. Sketch its graph.
2. Given the implicitly defined formula of a hyperbola, report its formula in parametric form.

Lesson: *Writing the Formula of a Hyperbola Given Its Graph*

Overview

We have seen that the parametric equations $x = f(t) = RUN\sec(t) + H$ and $y = g(t) = RISE\tan t) + K$ can be written implicitly as $\frac{(x-H)^2}{RUN^2} - \frac{(y-K)^2}{RISE^2} = 1$ if the axis through the vertices is horizontal and $x = f(t) = RUN\tan(t) + H$ and $y = g(t) = RISE\sec(t) + K$ can be written implicitly as $\frac{(y-K)^2}{RISE^2} - \frac{(x-H)^2}{RUN^2} = 1$ if the axis through the vertices is vertical. Given the sketch of the hyperbola we report its vertices and, once we have the center and RUN and RISE, we report the slope and equation of its asymptotes and its formula in implicit and parametric form. This material is in Section 14.4 in the text.

Learning Objectives

1. Given the graph of a hyperbola, report its center, vertices, RUN, RISE, and slopes and equations of the asymptotes.
2. Given the graph of a hyperbola, write a formula in implicit and parametric form.