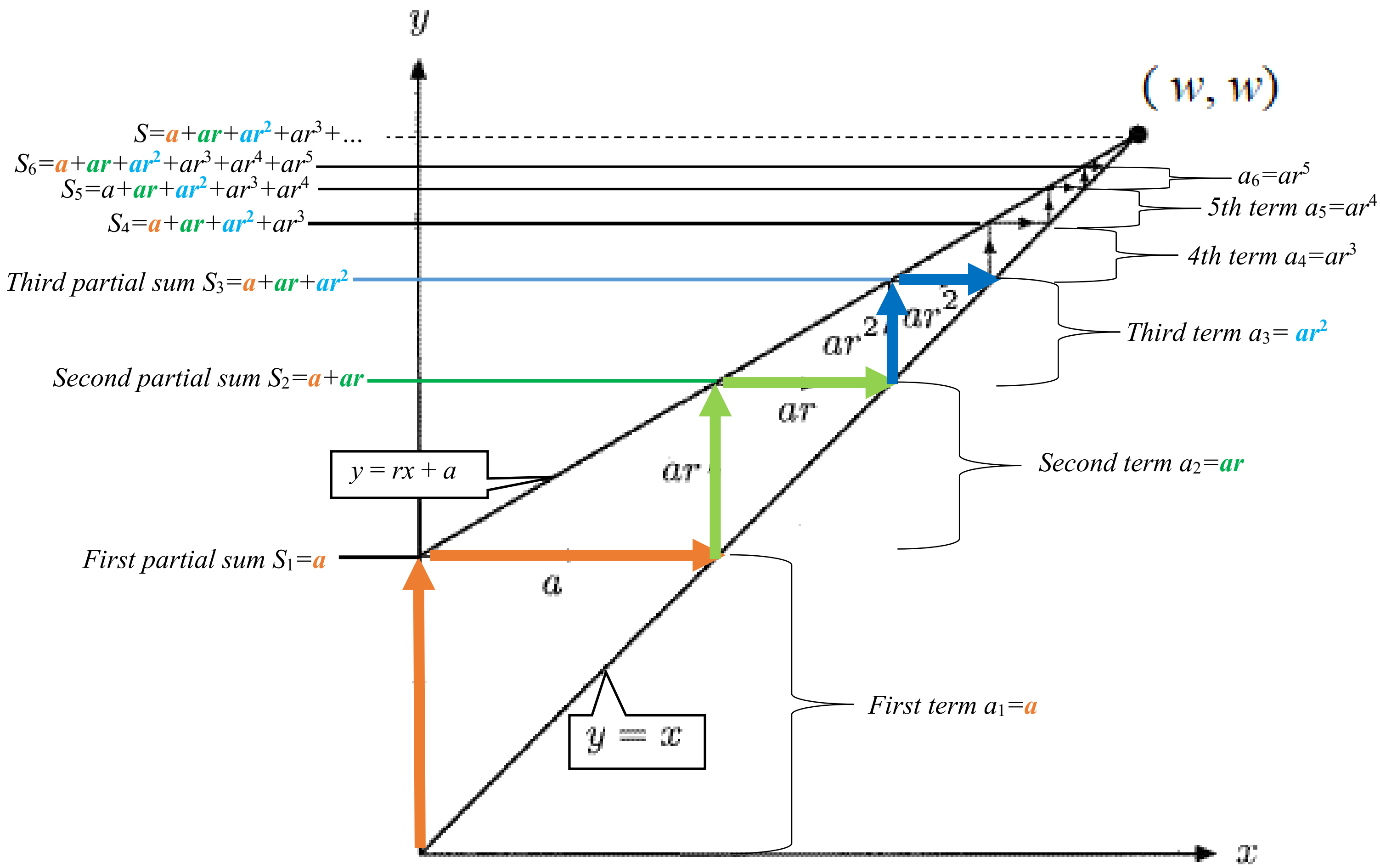
Visualizing the Sum of an Infinite Geometric Series

We have seen that if $0 < r < 1$ the geometric series $a + ar + ar^2 + ar^3 + \ldots$ converges to the point w, where w is the solution to the equation $rw + a = w$, which is $w = \dfrac{a}{1-r}$.

A way to visualize this convergence is as follows: Sketch the graphs of $y = rx + a$ and $y = x$.
Notice the point of intersection is the solution to $rx + a = x$.



Notice how the terms of the sequence a_n and the partial sums S_n appear in the above.
A series is said to converge to L when the sequence of partial sums converge to L. Assume $r > 0$. Complete.

1. When the slope of $y = rx + a$ is ________ $\{<, >, \geq, \leq\}$ the slope of $y = x$, then r _______ $\{<, >, \geq, \leq\}$ 1 and the lines "converge" at $w =$ ☐

   In this case we have $\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + \cdots = \dfrac{a}{1-r}$

   If instead of $k = 1$, we started at $k = 4$, i.e., $\sum_{k=4}^{\infty} ar^{k-1} = ar^3 + ar^4 + ar^5 \cdots$, what would change? Is L independent of the seed value in the recursive process?

   ____

2. When the slope of $y = rx + a$ is ________ $\{<, >, \geq, \leq\}$ the slope of $y = x$, then r _______ $\{<, >, \geq, \leq\}$ 1 and the lines ________________________.

   In this case we have $\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + \cdots = \infty$