Visualizing Terms and Partial Sums of Series

For each of the series $\sum_{k=1}^{\infty} a_k$, inspect several partial sums $S_n = \sum_{k=1}^{n} a_k$. Then conjecture if $\sum_{k=1}^{\infty} a_k$ converges.

1. Complete the last rows. If we merely scroll the table, it looks like $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} 1$ probably ______________ .
{converges, diverges}

Number of terms, n	nth term, $a_n = 1$	nth partial sum, $S_n = \sum_{k=1}^{n} 1$
1	$a_1 = 1$	$S_1 = 1$
2	$a_2 = 1$	$S_2 = 1+1 = 2$
3	$a_3 = 1$	$S_3 = 1+1+1 = 3$
4	$a_4 =$	$S_4 =$
⋮	⋮	⋮
100	$a_{100} = \boxed{}$	$S_{100} = \boxed{}$
⋮	⋮	⋮
m	$a_m = \boxed{}$	$S_m = \boxed{}$

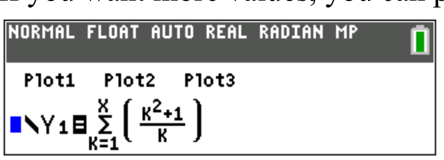
2. Complete the last row. If we merely scroll the table, it looks like $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} 2k$ probably ______________ .
{converges, diverges}

Number of terms, n	nth term, $a_n = 2n$	nth partial sum, $S_n = \sum_{k=1}^{n} 2k$
1	$a_1 = 2$	$S_1 = 2$
2	$a_2 = 4$	$S_2 = 2+4 = 6$
3	$a_3 = 6$	$S_3 = 2+4+6 = 12$
4	$a_4 = \boxed{}$	$S_4 = \boxed{}$

3. Complete the last row. If we merely scroll the table, it looks like $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{k^2+1}{k}$ probably ______________.
{converges, diverges}

Number of terms, n	nth term, $a_n = \frac{n^2+1}{n}$	nth partial sum, $S_n = \sum_{k=1}^{n} \frac{k^2+1}{k}$
1	$a_1 = \frac{2}{1} = 2$	$S_1 = 2$
2	$a_2 = \frac{5}{2}$	$S_2 = \frac{9}{2} = 4.5$
3	$a_3 = \frac{10}{3}$	$S_3 = \frac{47}{6} \approx 7.8333$
4	$a_4 = \boxed{}$	$S_4 = \frac{145}{12} \approx 12.0833$

If you want more values, you can put the expression $\sum_{k=1}^{x} \frac{k^2+1}{k}$ in Y1, then view a table.



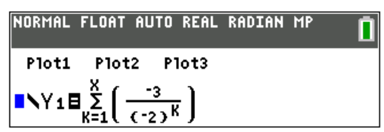
4. What is true about $\lim_{n\to\infty} a_n$ for each of these three series on this page? Does this affect $\lim_{n\to\infty} S_n$?

5. Complete the last row. If we merely scroll the table, it looks like $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{1-\cos \pi k}{k}$ probably ____________ .
{converges, diverges}

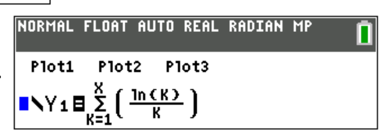
Number of terms, n	nth term, $a_n = \frac{1-\cos \pi n}{n}$	nth partial sum, $S_n \sum_{k=1}^{n} \frac{1-\cos \pi k}{k}$
1	$a_1 = \frac{2}{1} = 2$	$S_1 = 2$
2	$a_2 = 0$	$S_2 = 2$
3	$a_3 = \frac{2}{3}$	$S_3 = \frac{8}{3}$
4	$a_4 = 0$	$S_4 = \frac{8}{3}$
5	$a_5 = \frac{2}{5}$	$S_5 = \frac{46}{15}$
6	$a_6 = 0$	$S_6 = \frac{46}{15}$
⋮	⋮	⋮
99	$a_{99} = \boxed{}$	$S_{99} = \frac{6400711399252571342562758751832284129928}{1089380862964257455695840764614254743075}$

6. Complete the last rows. If we merely scroll the table, it looks like $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{-3}{(-2)^k}$ probably ____________ .
{converges, diverges}

Number of terms, n	nth term, $a_n \frac{-3}{(-2)^n}$	nth partial sum, $S_n = \sum_{k=1}^{n} \frac{-3}{(-2)^k}$
1	$a_1 = \frac{3}{2}$	$S_1 = \frac{3}{2}$
2	$a_2 = -\frac{3}{4}$	$S_2 = \frac{3}{4}$
3	$a_3 = \frac{3}{8}$	$S_3 = \frac{9}{8}$
4	$a_4 = -\frac{3}{16}$	$S_4 = \frac{15}{16}$
5	$a_5 = \frac{3}{32}$	$S_5 = \frac{33}{32}$
6	$a_6 = -\frac{3}{(\quad)}$	$S_6 = \frac{(\quad)}{(\quad)}$
7	$a_7 = \frac{3}{(\quad)}$	$S_7 = \frac{(\quad)}{128}$



You can put the expression $\sum_{k=1}^{x} \frac{-3}{(-2)^k}$ in Y1, then view a table.



7. If we merely scroll the table, it looks like $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{\ln k}{k}$ probably ____________ .
{converges, diverges}

Number of terms, n	nth term, $a_n = \frac{\ln n}{n}$	nth partial sum, $S_n = \sum_{k=1}^{\infty} \frac{\ln k}{k}$
1	$a_1 = \frac{\ln 1}{1} = 0$	$S_1 = 0$
2	$a_2 = \frac{\ln 2}{2}$	$S_2 = \frac{\ln 2}{2} \approx 0.3466$
3	$a_3 = \frac{\ln 3}{3}$	$S_3 = \frac{\ln 72}{6} \approx 0.7128$
4	$a_4 = \frac{\ln 4}{4}$	$S_4 = \frac{\ln 24}{3} \approx 1.0594$

8. What is true about $\lim_{n\to\infty} a_n$ for each of these three series on this page? Does this affect $\lim_{n\to\infty} S_n$?

9. Click **HERE** for four additional examples, two of which are very famous.

10. Rhino Bonus

The infinite series of the reciprocals of the squares of positive integers was known to converge, but no one could find the exact value until 1735, when Euler showed that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\frac{1}{7^2}+\cdots = \frac{\pi^2}{6}$.

The mathematical world rejoiced. See below why this discovery was a gift that kept on giving.

(+0.5) **a.** Use the fact that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\frac{1}{7^2}+\cdots = \frac{\pi^2}{6}$ to find the exact value of

$\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\frac{1}{8^2}+\cdots$

Show work below.

(+0.5) **b.** Use the fact that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\frac{1}{7^2}+\cdots = \frac{\pi^2}{6}$ and part **a** to find the exact value of

$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots$

Show work below.