Using Calculus with Series

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + ar^4 + \ldots = \frac{a}{1-r} \text{ if } |r| < 1$$

BIG DADDY

1. Consider $f(w) = \sum_{n=0}^{\infty} w^n = 1 + w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 + w^9 + w^{10} + w^{11} + \ldots$

a. Replace w with $-x^2$ in the above expression:

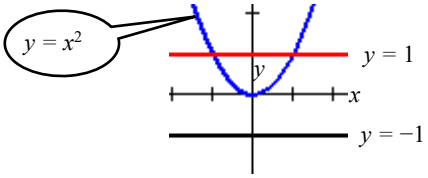
i. $f(-x^2) = \sum_{n=0}^{\infty} \left(\boxed{-x^2}\right)^n = 1 + \boxed{-x^2} + \left(\boxed{-x^2}\right)^2 + \left(\boxed{-x^2}\right)^3 + \left(\boxed{-x^2}\right)^4 + \ldots$

ii. Let's call the above child series $a(x)$. Simplify by removing all parentheses.

$a(x) = 1 + \boxed{-x^2} + \boxed{} + \boxed{} + \boxed{} + \ldots$

iii. Write the expression in sigma notation so it is simplified. $a(x) = \sum_{n=0}^{\infty} (-1)^n \boxed{\phantom{x^{2n}}}$

b. Report the radius of convergence of $a(x)$. **Tip:** Replace $r = w$ with $-x^2$ in the inequality $|r| < 1$. Since $y = |-x^2| = x^2$, you can solve this graphically finding when the parabola $y = x^2$ is below the line $y = 1$.



Complete: the radius of convergence of the series $a(x)$ is $R = \boxed{}$.

Explore the **left** endpoint. Use your expanded form of the series in **a.ii.**

Complete: $a(\boxed{}) = 1 + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \ldots = \boxed{}$

Therefore, the **left** endpoint of the interval of convergence, $x =$ _____ , is _____ in the interval.
{ included, excluded}

So on the graph of $a(x)$ at the left endpoint there is a _____
{ hole, point on the graph}

Explore the **right** endpoint. Use your expanded form of the series in **a.ii.**

Complete: $a(\boxed{}) = 1 + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \ldots = \boxed{}$

Therefore, the **right** endpoint of the interval of convergence, $x =$ _____ , is _____ in the interval.
{ included, excluded}

So on the graph of $a(x)$ at the right endpoint there is a _____.
{ hole, point on the graph}

We conclude the interval of convergence of the series $a(x)$ in part **1a** is the following.
Circle one and complete the blanks to the choice you selected.

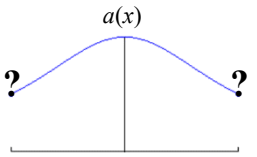
A. $\boxed{} \le x \le \boxed{}$ B. $\boxed{} < x \le \boxed{}$ C. $\boxed{} \le x < \boxed{}$ D. $\boxed{} < x < \boxed{}$

c. Within its interval of convergence, the series $a(x)$ in **1a** is equivalent to the closed form of the function below:

$$a(x) = \frac{1}{\boxed{}}$$

Tip: Replace $r = w$ with $-x^2$ in the closed form of yo' Big Daddy in the box at the top of the page.

d. Enter your closed form expression from part **1c** in a grapher using a viewing window where Xmin and Xmax would match your interval in part **1b**.
You should see the graph to the right. The graph has even symmetry.
Why would you expect this to be the case given what you wrote for part **1a.ii** and part **1c**?



Notice the *even* symmetry
(about the *y-axis*)

Fun Facts: The Genetics of Power Series!

We can create a new child series using

- **Algebraic Operations** (Substitution, Scaling, or Shifting):
  The child series can end up with a completely different **Center, Radius** and **Interval of Convergence** than its parent.
- **Calculus Operations** (Differentiation or Integration): The **Center** and **Radius of Convergence** are locked in and will match the parent series exactly.

The Endpoint Catch: While the radius survives calculus operations, the **Interval boundaries** might not.
Taking a *derivative* can cause an *included* endpoint to become *excluded*. Taking an *integral* may introduce an alternator, which can flip an *excluded* endpoint to *included* thanks to the Alternating Series Test. Always test your endpoints!