The Work to Build a Pyramid for Ramses Senior

Volume of slice $\approx s^2\Delta h$

481 ft
756 ft
756 ft
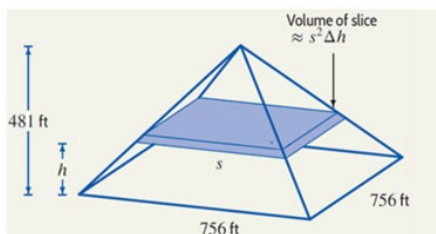
Ramses Senior has decreed that his pyramid shall be constructed of sandstone, $\rho = 200\ \frac{lb}{ft^3}$ as described above.

"So let it be written. So let it be done."

h	s
0	____
481	____
½ (481)	____

1. Complete the table and fill in the blanks.
    If $h = 0$ ft, then the side length, s, of a cross-section is s = _____ ft.
    If $h = 481$ ft, then the side length, s, of a cross-section is s = _____ ft.
    Report a formula that relates s to h. A graph may help.

    s = _____   Check: If $h = 240.5$ ft = ½ (481) (half the total height), then s = _____ ft.

2. Report the **weight** $F(h)$ of a thin, square s ft by s ft cross sectional slice of height h and thickness Δh

    $F(h)$ = _____

3. The very bottom layer at $h = 0$ (ground level) will need to be lifted a distance of 0 ft.
    The layer $h = 10$ ft high will need to be lifted a vertical distance of _____ ft.
    The layer $h = 240.5$ ft high will need to be lifted a vertical distance of _____ ft.
    The very top layer at $h = 481$ ft need will have be lifted a vertical distance of _____ ft.
   In general, the distance any slice that is h feet off the ground will need to be lifted a vertical distance of _____ ft.

4. Set up the integral that gives the total work done, in foot-pounds, to build the total pyramid.

5. Calculate the total work done to build the pyramid. $W \approx$ _____ x 10^{12} ft lb.

6. Ramses Senior thinks that at $h = 240.5$ ft (half the height), then half the work will be done.
    Do you agree? Explain.

    240.5 ft
    s
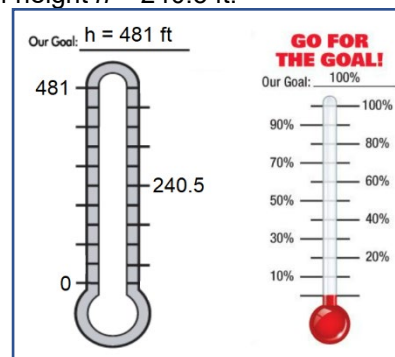   756 ft
    756 ft
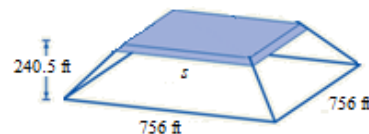
7. Set up and evaluate an integral to calculate the work done to build a truncated pyramid of height $h = 240.5$ ft.

    What percent of the total work has been completed?

8. Nephretiri is commanded to construct and maintain a progress thermometer.
    She decides to do two, as shown to the right.
    Shade both thermometers for the halfway point at $h = 240.5$.

    Our Goal: h = 481 ft
    481
    240.5
    0

    GO FOR THE GOAL!
    Our Goal: 100%
    100%
    90%
    80%
    70%
    60%
    50%
    40%
    30%
    20%
    10%

9. Construct a graph of $W'(h)$, the rate at which the work is completed at height h.
    At what height is her thermometer rising the fastest?

10. (+0.5 Rhino Bonus): It is reported the pyramid took 20 years to build. Let's assume every laborer worked 10 hours a day, 300 days a year, for 20 years of their life. Assume that a typical worker lifted ten 50-pound blocks a distance of 4 feet every hour, thus performing 2000 foot-pounds of work per hour (this is a very rough estimate). Find how much work each laborer performed over a 20 year period and then estimate the number of workers needed to build the pyramid.