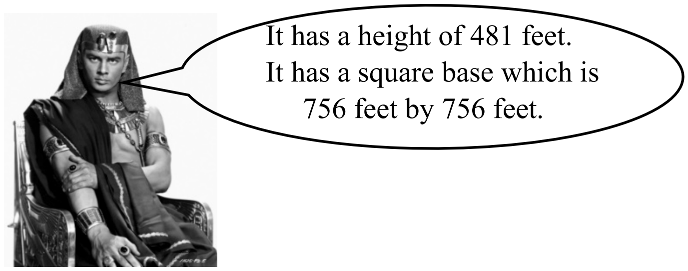
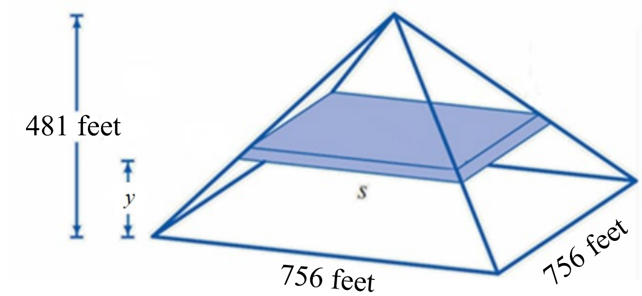
The Volume of a Pyramid

When it was built more than 4500 years ago, the Great Pyramid of Giza in Cairo, Egypt, had the dimensions below.



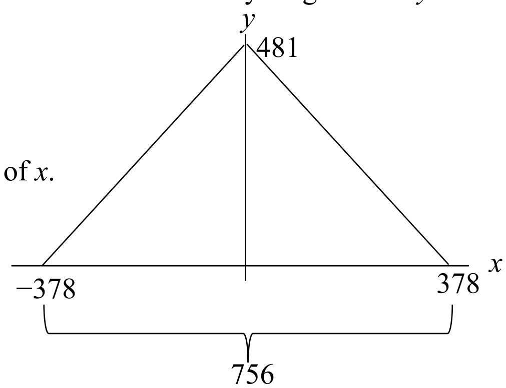


1. First, examine two extreme cases. What is the side length, s, of a square cross-sectional slice if
   a. $y = 0$?

   b. $y = 481$?

2. We want a function which relates the length, s, of a square cross-sectional slice to any height value y.

METHOD 1:



a. Write y in terms of x for the decreasing line.

$y =$ ______________________________

b. Write the length s, of a square cross-sectional slice in terms of x.

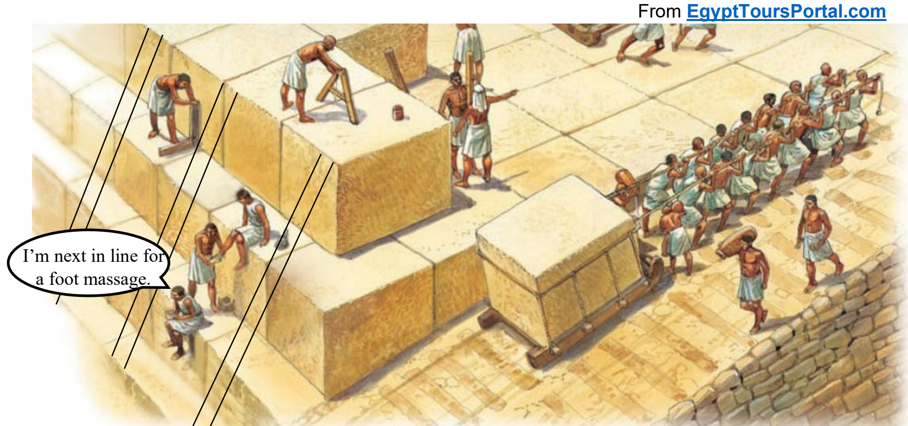
$s =$ ______________________________

c. Write y in terms of s.

$y =$ ______________________________

d. Write s in terms of y.

From EgyptToursPortal.com



METHOD 2:

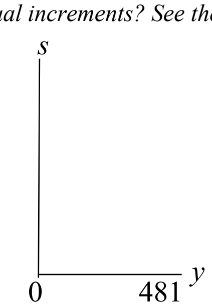
To find a function which relates the length, s, of a square cross-sectional slice and any height y, use Question 1.

a. Plot the points from Question 1 that represent the two extremes on the axes below.

b. Complete: as y increases, s should decrease. How? ____.

{at a **constant** rate, at an **increasing** rate, at a **decreasing** rate}

If the height increases in equal increments, does the length, s, of the next layer decrease in equal increments? See the image above.



c. Sketch the graph of s vs. y on the axes. Label the vertical axis with a number.

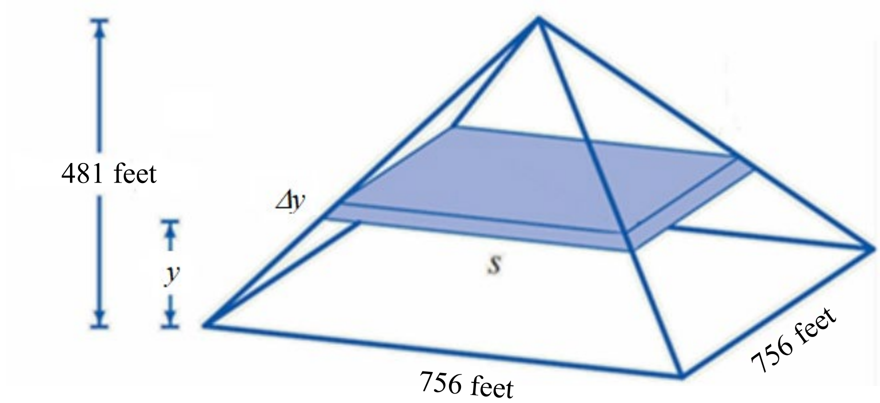
d. Write a formula which gives s for any value of y.

TIP: Check that your formula matches what you found in Question 1a and 1b.

3. In Question 2, which method did you prefer? (A third method is also possible using similar triangles.)

4. Report the cross-sectional area $A(y)$ of one representative square slice of pyramid .

   $A(y) =$ ____________________



5. A representative square slice with thickness Δy and cross-sectional area s^2 has volume = ____________________

6. Set up the definite integral that gives the volume of the pyramid.

$$V = \int_{\boxed{0}}^{\boxed{}} \left(\boxed{} \right) dy$$

involves y

7. Find the value of your integral using FNINT on your calculator to compute the volume, in cubic feet. Check that it matches the value that you would get if you used the formula for the volume of a pyramid below told by Ramses II, where b is the length of the base and h is the height.

   Volume = ____________________ cubic feet.

Rhino Participation Bonus +1: Replace your value of 481 with h and your value of 756 with b in Question 6.

$$V = \int_{\boxed{0}}^{\boxed{}} \left(\boxed{} \right) dy$$

(The boxes contain expressions involving ***only*** *h and b.)*

Then use the FTC to find the antiderivative of the above to show, in general, the volume of a pyramid is, indeed, $V = \frac{1}{3}b^2h$. Correct work using proper notation is required for credit.