The Radius of Convergence

Suppose we extended series to complex numbers.

1. Consider the series $f(z) = \sum_{n=0}^{\infty} 10z^n = 10 + 10z + 10z^2 + 10z^3 + 10z^4 + 10z^5 + 10z^6 + 10z^7 + 10z^8 + \ldots$

a. If $z = 1$, the series for $f(z)$ ________________________________. Explain.
{converges, does not exist, approaches $-\infty$, approaches ∞}

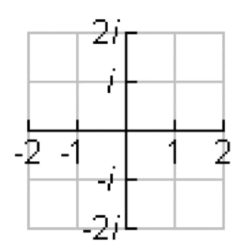
b. If $z = -1$, the series for $f(z)$ ________________________________. Explain.
{converges, does not exist, approaches $-\infty$, approaches ∞}

c. If $z = i$, the series for $f(z)$ ________________________________. Explain.
{converges, does not exist, approaches $-\infty$, approaches ∞}

d. If $z = -i$, the series for $f(z)$ ________________________________. Explain.
{converges, does not exist, approaches $-\infty$, approaches ∞}

e. If $z = \frac{1}{2}i$, the series for $f(z)$ ________________________________. Explain.
{converges, does not exist, approaches $-\infty$, approaches ∞}

f. Use the ratio test to determine the **interval** of convergence.



g. Sketch the graph of the points inside the interval of convergence on the complex plane.

h. Report the **radius** of convergence. $R =$ _____

i. Why do they call it the radius of convergence?

j. What kind of series is $f(z) = \sum_{n=0}^{\infty} 10z^n = 10 + 10z + 10z^2 + 10z^3 + 10z^4 + 10z^5 + 10z^6 + 10z^7 + 10z^8 + \ldots$?

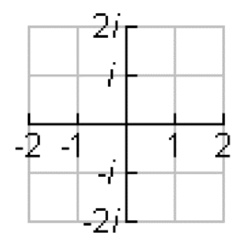
k. On its interval of convergence, the infinite series for $f(z)$ is equivalent to the rational function $f(z) =$

l. Confirm this with your calculator in complex mode for $f\left(\frac{i}{2}\right)$.

2. Consider the series $f(z) = \sum_{n=1}^{\infty} n!z^n = 1!z + 2!z^2 + 3!z^3 + 4!z^4 + 5!z^5 + 6!z^6 + 7!z^7 + 8!z^8 + \ldots$

a. Use the ratio test to determine the **interval** of convergence.

b. Sketch the graph of the points inside the interval of convergence on the complex plane.



c. Report the **radius** of convergence. $R =$ _____