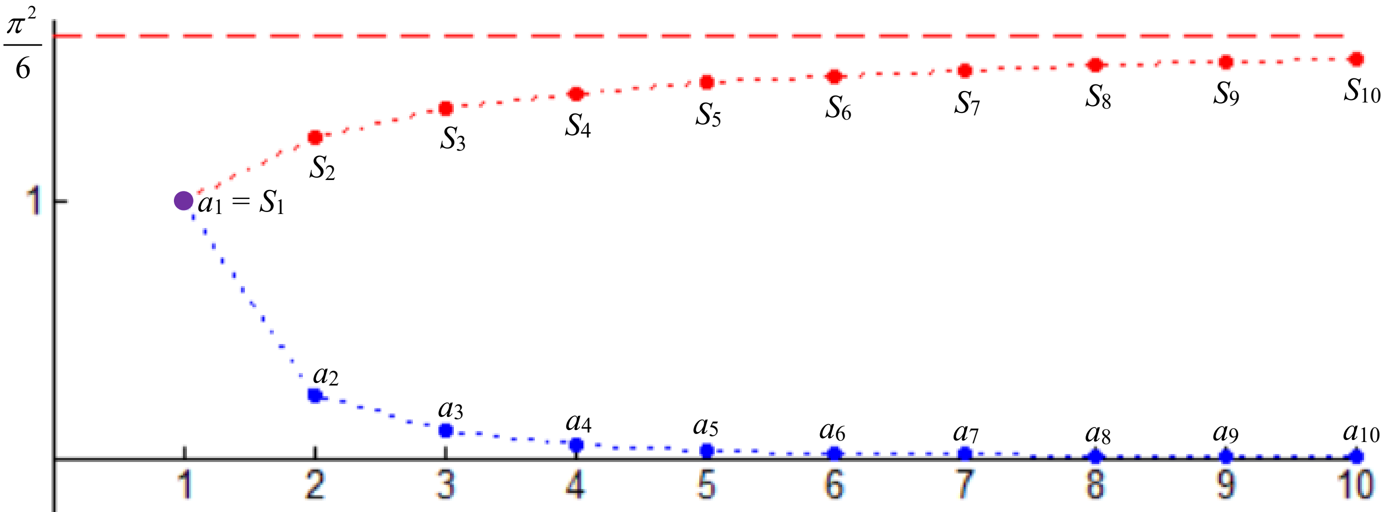
Rhino Bonus: The Sum of $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{1}{k^2}$ and the Sum of $\sum_{k=1}^{\infty} (-1)^{k+1} a_k = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$

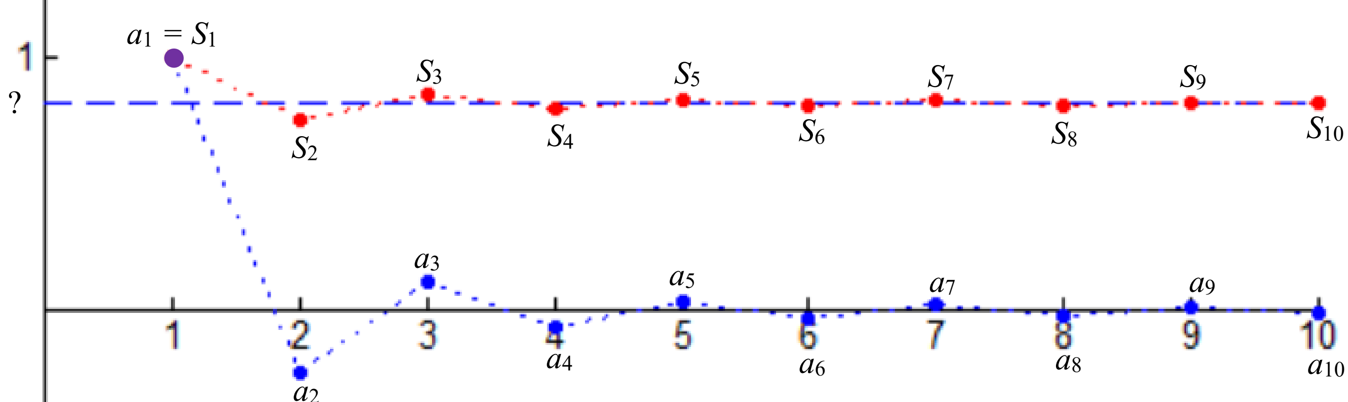


In 1735, Euler showed this amazing fact: $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{6}$.

Above is a plot of the terms, (n, a_n), along with the plot of the nth partial sums (n, S_n).

(+1) Use Euler's result that $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{6}$ to determine the **exact** value of the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} a_k = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \cdots$

Below is a plot of the terms, (n, a_n), along with the plot of the nth partial sums (n, S_n) for the alternating series. Notice it converges much faster than the series of original unsigned positive terms.



To check your answer, you can produce the above graphs of the partial sums on your calculator.

Hint: the values of $\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \frac{1}{8^2} + \cdots$ and $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$ might be helpful.

Show work clearly on the back for credit, working with

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \frac{1}{7^2} - \frac{1}{8^2} + \cdots$ in its expanded form (also called long form).