MA 16600 Practice Questions over 12.2-12.3

1. Write a pair of polar coordinates (r, θ) and a pair of rectangular coordinates (x, y) for the points **A** through **I**. ***Give exact values. Report θ in radians please.*** Utilize the unit circle for efficiency. No trig function or decimal answer should be reported. Only one polar coordinate (of your choice) need be reported.
For θ please choose from 0, π/6, π/4, π/3, π/2, 2π/3, 3π/4, 5π/6, π, 7π/6, 5π/4, 4π/3, 3π/2, 5π/3, 7π/4, or 11π/6

A. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____

B. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____

C. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____

D. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____

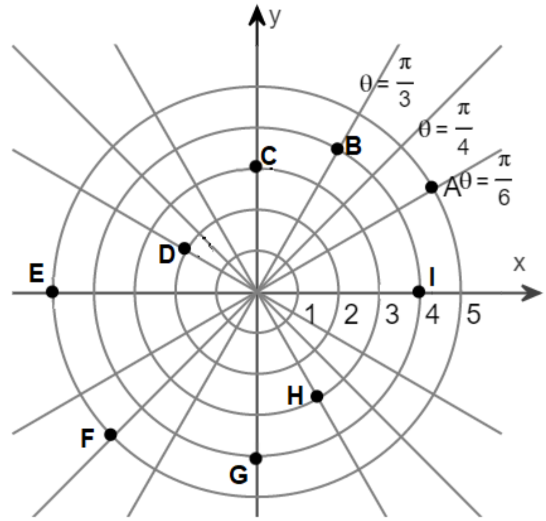
E. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____

F. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____

G. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____

H. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____

I. $r =$ ____, $\theta =$ ______ and $x =$ _____, $y =$ _____



2. Given the polar equation in terms of r and θ, write the Cartesian equation in terms of x and y. Your equation should begin with "$y =$". Sketch a graph of the curve.

a. $2r = \csc\theta$ **b.** $r = \dfrac{\tan\theta}{\cos\theta - \sin\theta}$ **c.** $r = \dfrac{1}{\cos\theta + \sin\theta}$ **d.** $r\cot\theta + r^2\cos\theta = 2\csc\theta$

e. $r = 3\csc\theta(1 - \tan\theta)$ **f.** $r^2\cos\theta + r\tan\theta = \sec\theta$ **g.** $r^2 = \sec^2\theta\tan\theta$ **h.** $r = \dfrac{2\csc\theta}{\cot\theta + r\cos\theta}$

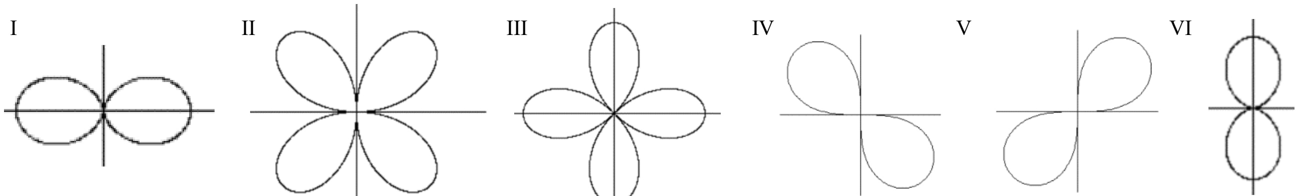
3. Given the Cartesian equation in terms of x and y, write the polar equation in terms of r and θ. Your equation should begin with "$r =$". Sketch a graph of the curve.

a. $x^2 + y^2 = 6x - 4y$ **b.** $x^2(x^2 + y^2) = 16y^2$ **c.** $y = 3 - 2x$

4. Given the Cartesian equation in terms of x and y, write the polar equation in terms of r and θ for each. Your equation should begin with "$r =$". Then use your polar grapher to match it with the corresponding graph from the rose bouquet.

a. $(x^2 + y^2)^2 = 128xy$ **b.** $(x^2 + y^2)^3 = 64x^4$ **c.** $(x^2 + y^2)^3 = 64y^4$

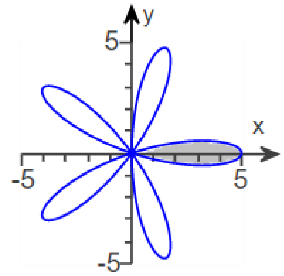
ROSE BOUQUET:



5. The area from $\theta = \alpha$ to $\theta = \beta$ inside a polar graph is $\int_\alpha^\beta \frac{1}{2}r^2\,d\theta$.
Also use your grapher (not calculus) to report the exact area of the region. Report your answer as a multiple of π.

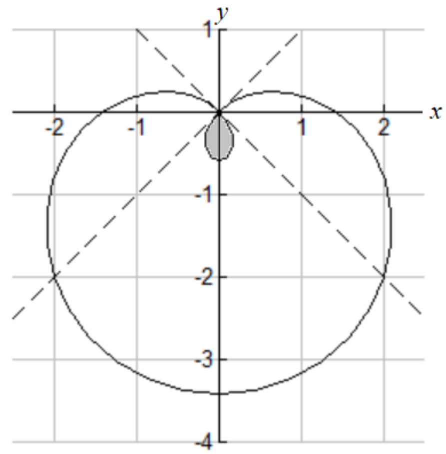
a. $r = 4 - 4\sin\theta$ **b.** $r = 6 - 5\cos\theta$

6. The area from $\theta = \alpha$ to $\theta = \beta$ inside a polar graph is $\int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$.



a. Find the exact area of the region inside one leaf of the 5-leaved rose $r = 5\cos 5\theta$
You can use the FNINT command, but provide an exact area.

$\int_{\square}^{\square} \square \, d\theta = \square$



b. Set up the integral to calculate the area of the region inside the inner loop of the limaçon $r = \sqrt{2} - 2\sin\theta$. Use the FNINT command to find the area and approximate it the area to two decimal places.

To find the integration limits, find where $r = \sqrt{2} - 2\sin\theta = 0$ where $0 \le \theta < 2\pi$, since this will be where the inner loop starts and ends.

TIP: The dashed lines in the above graph are the polar equations $\theta = \alpha$ and $\theta = \beta$, where α and β are the lower and upper limits of integration. You can enter these values in your polar grapher as θmin and θmax to check that you have sketched only the inner loop.

$\int_{\square}^{\square} \square \, d\theta \approx \square$