MA 16600 Practice Questions over 8.3 and Appendix C

**1.** State the double-angle identity used to integrate $\sin^2 x$. $\sin^2 x =$ ______

**2.** State the double-angle identity used to integrate $\cos^2 x$. $\cos^2 x =$ ______

**3.** Find the indefinite integral. Show work. $\int \sin^3 x \, dx$

**4.** Find the indefinite integrals. Show work.

**a.** $\int \tan^9 x \sec^2 x dx$

**b.** $\int \cos^2 \theta d\theta$

**c.** $\int \sin^3 x \cos^6 x dx$

**5.** Consider the integral $\int \frac{\sin\theta}{\cos^2\theta} d\theta$. Which of the following

A. $\sin\theta + C$ B. $\cos\theta + C$ C. $\tan\theta + C$ D. $\csc\theta + C$ E. $\sec\theta + C$ F. $\cot\theta + C$
G. $-\sin\theta + C$ H. $-\cos\theta + C$ I. $-\tan\theta + C$ J. $-\csc\theta + C$ K. $-\sec\theta + C$ L. $-\cot\theta + C$
M. All of these N. None of these.

**6.** Consider $\int \sec^{14} x \tan^{17} x \, dx$

**a.** Suppose we let $u = \tan x$. Then $du =$ ________________ $dx$

Then we can write $\int \sec^{14} x \tan^{17} x \, dx = \int$ ______ $du.$

Your answer is a binomial in terms of $u$ raised to a power multiplied by $u$ raised to a power.
Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial.

**b.** Suppose we let $w = \sec x$. Then $dw =$ ________________ $dx$

Then we can write $\int \sec^{14} x \tan^{17} x \, dx = \int$ ______ $dw.$

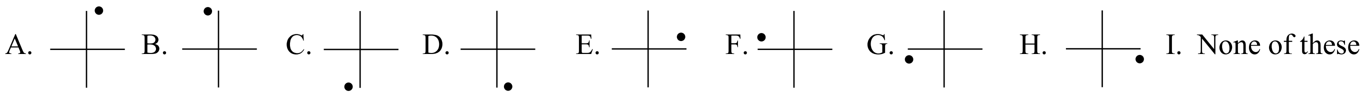
Your answer is a binomial in terms of $w$ raised to a power multiplied by $w$ raised to a power.
Do not multiply it out. Do not find the antiderivative. Just leave it as a polynomial.

**7. a.** Use your calculator to compute $z = \dfrac{5-i}{5+i}$ and report in the form $z = x + yi$, where $x$ and $y$ are **exact** real numbers.

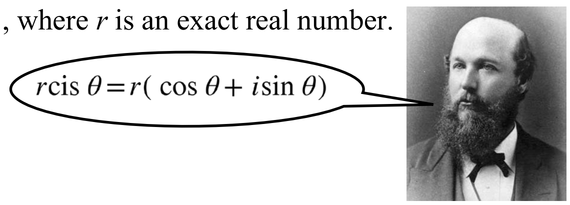
Then find the absolute value of $z$. $z = \dfrac{5-i}{5+i} = \square + \square\, i$

**b.** $|z| = \left|\dfrac{5-i}{5+i}\right| = \square$ (Use your calculator's absolute value command.)

**c.** Determine the location of $z$ on the complex plane. Assume a square grid. (Select one)




**d.** Write the complex number $z$ in polar form $r\text{cis }\theta$, where $r$ is an exact real number. Approximate $\theta$ in degrees to 1 decimal place.



Washington Irving Stringham

$z = \dfrac{5-i}{5+i} = \square \text{ cis } \square$

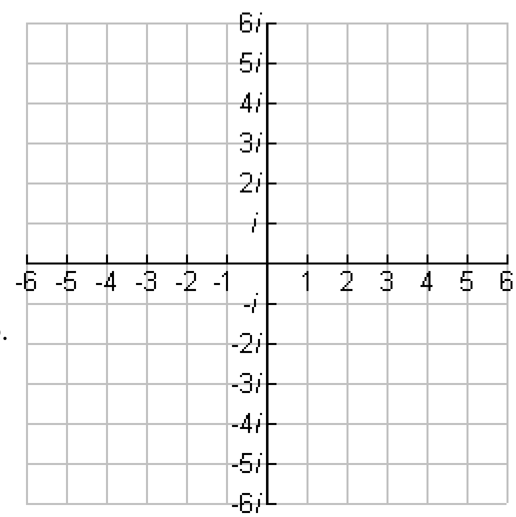
**8. a.** Plot and **label** the complex numbers on the grid shown.

$z_1 = 5 - 5i$

$z_2 = -3i$

$z_3 = 3\sqrt{2}\text{ cis }\dfrac{3\pi}{4}$

$z_4 = 5\text{cis }7\pi$




**b.** Write $z_1$ and $z_2$ in polar form $r\text{cis }\theta$, where $r$ and $\theta$ are exact real numbers (and $\theta$ is in radians). Hint: Part (a) may help. (There are many correct answers for $\theta$; however, report exact **radians** please.)

For $\theta$ please choose from 0, π/6, π/4, π/3, π/2, 2π/3, 3π/4, 5π/6, π, 7π/6, 5π/4, 4π/3, 3π/2, 5π/3, 7π/4, or 11π/6

$z_1 = 5 - 5i$

$r =$ ______________

$\theta =$ ______________

Polar form $r\text{cis}\,\theta$ of $z_1 = 5 - 5i = \square \text{ cis } \square$

$z_2 = -4i$

$r =$ ______________

$\theta =$ ______________

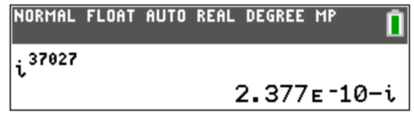
Polar form $r\text{cis}\,\theta$ of $z_2 = -4i = \square \text{ cis } \square$

**c.** Write $z_3$ and $z_4$ in rectangular form $x + yi$, where $x$ and $y$ are **exact** real numbers.

$z_3 = 3\sqrt{2}\text{ cis }\dfrac{3\pi}{4} = \square + \square \cdot i$

$z_4 = 5\text{cis }7\pi = \square + \square \cdot i$

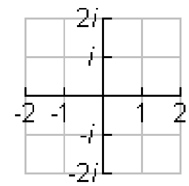
**9.** Consider the complex number $i^{37027}$.




**a.** A student uses a calculator to try to write the number in rectangular form $x + yi$, where $x$ and $y$ are real numbers. See the screen shown.
What should the exact answer really be? Report the exact answer in rectangular form $x + yi$:

$$i^{37027} = \boxed{\quad} + \boxed{\quad} \cdot i$$

**b.** Report the location of $i^{37027}$ in the complex plane. Plot the point.




A. the positive real axis  B. the positive imaginary axis  C. the negative real axis
D. the negative imaginary axis  E. none of these

**c.** Use part **b** to report $i^{37027}$ in polar form in polar form $r\text{cis}\,\theta$, where $r$ and $\theta$ are exact real numbers (and $\theta$ is in radians). Hint: Part (b) may help. (There are many correct answers for $\theta$; however for $\theta$ please choose from 0, π/6, π/4, π/3, π/2, 2π/3, 3π/4, 5π/6, π, 7π/6, 5π/4, 4π/3, 3π/2, 5π/3, 7π/4, or 11π/6)

Polar form $r\text{cis}\,\theta$ of $i^{37027} = \boxed{\quad}\text{cis}\boxed{\quad}$

**10.** Consider the complex geometric series $f(z) = \sum_{k=0}^{\infty} 50z^k = 50 + 50z + 50z^2 + 50z^3 + \ldots$ which converges on $|z| < 1$. Report the value of $f\left(\frac{3i}{4}\right) = \sum_{k=0}^{\infty} 50\left(\frac{3i}{4}\right)^k$.

**a.** We separate even powers of $\frac{3i}{4}$ and odd powers of $\frac{3i}{4}$.

$$f\left(\frac{3i}{4}\right) = \sum_{k=0}^{\infty} 50\left(\frac{3i}{4}\right)^k = 50\left(1 + \left(\frac{3i}{4}\right)^2 + \left(\frac{3i}{4}\right)^4 + \left(\frac{3i}{4}\right)^6 + \ldots\right) + 50\left(\left(\frac{3i}{4}\right)^1 + \left(\frac{3i}{4}\right)^3 + \left(\frac{3i}{4}\right)^5 + \left(\frac{3i}{4}\right)^7 + \ldots\right)$$

First simplify powers of $\boldsymbol{i}$. Then combine real parts in the first row and imaginary parts in the second row.
Then factor out 50 in the first row and $50 \cdot \frac{3i}{4}$ in the second row. Enter **real** numbers in each box.
You can write the real numbers as powers of $\frac{3}{4}$.

$$f\left(\frac{3i}{4}\right) = 50\left(1 + \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \ldots\right) + 50 \cdot \frac{3i}{4}\left(1 + \boxed{\quad} + \boxed{\quad} + \boxed{\quad} + \ldots\right)$$

**b.** The geometric series $1 + \left(\frac{3i}{4}\right)^2 + \left(\frac{3i}{4}\right)^4 + \left(\frac{3i}{4}\right)^6 + \ldots$ has $a = 1$ and $r = \boxed{\quad}$ and sum equal to $\boxed{\quad}$.

The geometric series here has $a = 1$ and $r = \boxed{\quad}$ and sum equal to $\boxed{\quad}$.

**c.** Combining, we have $f\left(\frac{3i}{4}\right) = \sum_{k=0}^{\infty} 50\left(\frac{3i}{4}\right)^k = \boxed{\quad} + \boxed{\quad}\, i$ (Insert integers in the boxes.)

**d.** What can be said about $f(i) = \sum_{k=0}^{\infty} 50(i)^k$?

A. The sum converges to the complex number $f(i)$ = _______ + _________$i$ (Insert real numbers in the blanks.)
B. The sum diverges to ∞

C. The sum diverges to $-\infty$.
D. The limit of the partial sums does not exist.