Practice Questions from Section 10.5 and Section 10.6

Insert numbers or expressions with the correct variables in the boxes. Circle the correct choice in the word bank.

1. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{7/4}}$.

a. We can use the Comparison Test with $b_n =$ ☐ to show the series $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{7/4}}$ ______ {converges, diverges}

b. Which is true for your choice of b_n ?

Circle one: **i.** $\frac{\sin^2 n}{n^{7/4}} \le b_n$ **ii.** $b_n \le \frac{\sin^2 n}{n^{7/4}}$

c. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______ {converge, diverge} because ______

(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as *p*-series, harmonic series, geometric series, etc.)

2. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\tan^{-1} n}{3^n}$.

a. We can use the Comparison Test with $b_n =$ ☐ to show the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{3^n}$ ______ {converges, diverges}

b. Which is true for your choice of b_n ?

Circle one: **i.** $\frac{\tan^{-1} n}{3^n} \le b_n$ **ii.** $b_n \le \frac{\tan^{-1} n}{3^n}$

c. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______ {converge, diverge} because ______

(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as *p*-series, harmonic series, geometric series, etc.)

3. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n}{n^2 - \cos^2 n}$.

a. We can use the Comparison Test with $b_n =$ ☐ to show the series $\sum_{n=1}^{\infty} \frac{n}{n^2 - \cos^2 n}$ ______ {converges, diverges}

b. Which is true for your choice of b_n ?

Circle one: **i.** $\frac{n}{n^2 - \cos^2 n} \le b_n$ **ii.** $b_n \le \frac{n}{n^2 - \cos^2 n}$

c. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______ {converge, diverge} because ______

(Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as *p*-series, harmonic series, geometric series, etc.)

4. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{6n^2+n+7}{n^5+2n}$.

i. We can use the Limit Comparison Test with $b_n =$ [ ] to show the series $\sum_{n=1}^{\infty} \frac{6n^2+n+7}{n^5+2n}$ ________ {converges, diverges}

ii. Complete, assuming b_n is what you wrote in the box in part **i**.

$\sum_{n=1}^{\infty} b_n$ will ________ {converge, diverge} because ________ (Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} =$ [ ].

5. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{5n^5+8n^2}{\sqrt{n^{12}+8n^2}}$.

i. We can use the Limit Comparison Test with $b_n =$ [ ] to show the series $\sum_{n=1}^{\infty} \frac{5n^5+8n^2}{\sqrt{n^{12}+8n^2}}$ ________ {converges, diverges}

ii. Complete, assuming b_n is what you wrote in the box in part **i**.

$\sum_{n=1}^{\infty} b_n$ will ________ {converge, diverge} because ________ (Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} =$ [ ].

6. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=4}^{\infty} \frac{1}{7\sqrt{n^3-4n+16}}$.

i. We can use the Limit Comparison Test with $b_n =$ [ ] to show the series $\sum_{n=4}^{\infty} \frac{1}{7\sqrt{n^3-4n+16}}$ ________ {converges, diverges}

ii. Complete, assuming b_n is what you wrote in the box in part **i**.

$\sum_{n=1}^{\infty} b_n$ will ________ {converge, diverge} because ________ (Give a reason for your answer on how you know $\sum b_n$ converges or diverges, such as p-series, harmonic series, geometric series, etc.)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} =$ [ ].

7. Suppose $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = \frac{2}{1} - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \frac{6}{5} - \frac{7}{6} + \ldots$

a. What is a_n? $a_n =$ [ ]

b. The series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ will ________________. {converge absolutely, converge conditionally, diverge }

c. Give a reason for your claim in part **b**.

8. Each alternating series below converges by the Alternating Series Test (AST).
Determine if the convergence is conditional or absolute.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n 7n}{4n^3 - 3}$ will converge ________________ because
{ absolutely, conditionally }

the series $\sum_{n=1}^{\infty} \frac{7n}{4n^3 - 3}$ will ________________ by the ______________________________ with b_n = [ ]
{converge, diverge} {Comparison Test, Limit Comparison Test}

Provide the details of your claim below.

b. $\sum_{n=1}^{\infty} (-1)^{n+1} a_n = \frac{1}{15} - \frac{1}{20} + \frac{1}{25} - \frac{1}{30} + \frac{1}{35} - \frac{1}{40} + \ldots$ will converge ____________________ because
{ absolutely, conditionally }

the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}$ [ ] will ________________ by the ______________________________ with b_n = [ ].
{converge, diverge} {Comparison Test, Limit Comparison Test}

Provide the details of your claim below.

9. Report the two conditions for an alternating series $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} (-1)^{n+1} a_n$ to converge, where a_n is positive for all n.

i. ____

ii. ____

10. Give an example of a divergent alternating series with the property that its nth term approaches 0.
There are many correct answers. Hint: think of your answer to Question **9**.
You may write it in long form (expanded form) or use sigma notation, but use correct notation.