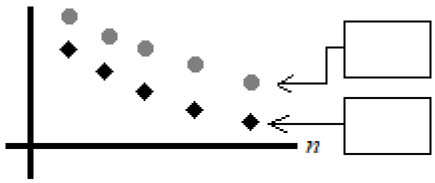
Practice Questions from Section 10.5

1. Complete the boxes and blanks. You can just circling the correct choices in the word bank.

a. **i.** To use the Direct Comparison Test to show the series $\sum_{n=1}^{\infty} c_n$ diverges,

we find a ______________ series $\sum_{n=1}^{\infty} d_n$ and must show that, for some large enough n, we have c_n ______ d_n.
{convergent, divergent} $\{\leq, \geq\}$

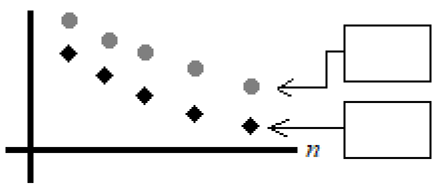
ii. Below is a plot of (n, c_n) and (n, d_n). Insert in the box which is c_n and which is d_n.



b. **i.** To use the Direct Comparison Test to show the series $\sum_{n=1}^{\infty} c_n$ converges,

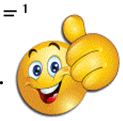
we find a ______________ series $\sum_{n=1}^{\infty} d_n$ and must show that, for some large enough n, we have c_n ______ d_n.
{convergent, divergent} $\{\leq, \geq\}$

ii. Below is a plot of (n, c_n) and (n, d_n). Insert in the box which is c_n and which is d_n.



c. **i.** Suppose Finn decides to use the Direct Comparison Test to investigate the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

Complete the boxes. Finn chooses to compare $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ to the series $\sum_{n=1}^{\infty} 1$.

He tells you, "Since $\ln n < n$, then $\frac{\ln n}{n} < \frac{n}{n} = 1$" You respond: A.  B. 

Finn goes on to say "We know that $\sum_{n=1}^{\infty} 1 = \square$ because of the ________________________________."
(Give a reason for your answer such as harmonic series, p-series with $p > 1$, p-series with $p < 1$, geometric series, nth Term Test for Divergence, etc.)

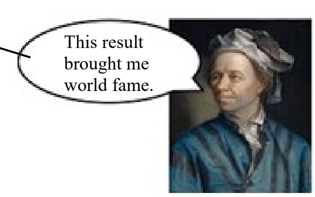
He concludes "Therefore by the Direct Comparison Test we know $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ diverges."

You respond: A.  B. 

ii. What alternative Convergence Test(s) could Finn use to show $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ diverges? ________________________
Select all possible answers.
(p-series with $p > 1$, p-series with $p < 1$, geometric series, Integral Test, nth Term Test)

d. **i.** Suppose Gil decides to use the Direct Comparison Test to investigate the series $\sum_{n=1}^{\infty} \left(\frac{n}{n^3 - n + 1}\right)$.

Complete the boxes. Gil compares $\sum_{n=1}^{\infty} \left(\frac{n}{n^3 - n + 1}\right)$ to the p-series $\sum_{n=1}^{\infty} \frac{1}{n^2} = \square$



Gil tells you, "Since $n^3 - n + 1 \le n^3$ for $n \ge 1$, then $\frac{1}{n^2} = \frac{n}{n^3} \le \frac{n}{n^3 - n + 1}$." You say: A.  B. 

Gil concludes "Therefore by the Direct Comparison Test we know $\sum_{n=1}^{\infty}\left(\frac{n}{n^3 - n + 1}\right)$ converges."

You respond: A.  B. 

ii. What alternative Convergence Test(s) could Gil use to show $\sum_{n=1}^{\infty}\left(\frac{n}{n^3 - n + 1}\right)$ converges? ________________

Select all possible answers. (Geometric series, Limit Comparison Test, Integral Test, *n*th Term Test for Divergence)

2. Consider the series $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{7/4}}$. Complete the boxes and blanks.

a. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{7/4}}$ will ______________ {converge, diverge}

b. Complete the box: We can use the Direct Comparison Test with $\sum_{n=1}^{\infty} \boxed{}$ to validate our claim in part **a**.

c. Assuming the contents of the dashed box are same as the *n*th term of your series in part **b**, which choice is true?

Circle one and complete the box: **i.** $\frac{\sin^2 n}{n^{7/4}} \le \boxed{}$ **ii.** $\boxed{} \le \frac{\sin^2 n}{n^{7/4}}$

d. Complete, assuming the series in tbe dashed box below is what you wrote in the box in part **b**.

$\sum_{n=1}^{\infty} \boxed{}$ will __________ {converge, diverge} because ____

(Give a reason for your answer such as harmonic series, *p*-series with $p > 1$, *p*-series with $p < 1$, geometric series, *n*th Term Test for Divergence)

3. Consider the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{3^n}$. Complete the boxes and blanks.

a. $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{3^n}$ will ______________ {converge, diverge}

b. Complete the box: We can use the Direct Comparison Test with $\sum_{n=1}^{\infty} \boxed{}$ to validate our claim in part **a**.

c. Assuming the contents of the box are same as the *n*th term of your series in part **b**, which choice is true?

Circle one and complete the box: **i.** $\frac{\tan^{-1} n}{3^n} \le \boxed{}$ **ii.** $\boxed{} \le \frac{\tan^{-1} n}{3^n}$

d. Complete, assuming the series in tbe box below is what you wrote in the box in part **b**.

$\sum_{n=1}^{\infty} \boxed{}$ will ______________ {converge, diverge} because ____

(Give a reason for your answer such as harmonic series, *p*-series with $p > 1$, *p*-series with $p < 1$, geometric series, *n*th Term Test for Divergence)

4. Consider the series $\sum_{n=1}^{\infty} \frac{n}{2n^2 - \cos^2 n}$. Complete the boxes and blanks.

a. $\sum_{n=1}^{\infty} \frac{n}{2n^2 - \cos^2 n}$ will ______________ {converge, diverge}

b. Complete the box: We can use the Direct Comparison Test with $\sum_{n=1}^{\infty}$ [ ] to validate our claim in part **a**.

c. Assuming the contents of the dashed box are same as the *n*th term of your series in part **b**, which choice is true?

Circle one and complete the box: **i.** $\frac{n}{2n^2 - \cos^2 n} \le$ [ ] **ii.** [ ] $\le \frac{n}{2n^2 - \cos^2 n}$

d. Complete, assuming the series in tbe dashed box below is what you wrote in the box in part **b**.

$\sum_{n=1}^{\infty}$ [ ] will __________ {converge, diverge} because ____

(Give a reason for your answer such as harmonic series, *p*-series with $p > 1$, *p*-series with $p < 1$, geometric series, *n*th Term Test for Divergence)

5. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{6n^2 + n + 7}{n^5 + 2n}$. Complete the boxes and blanks.

a We can use the Limit Comparison Test with $b_n =$ [ ] to show $\sum_{n=1}^{\infty} \frac{6n^2 + n + 7}{n^5 + 2n}$ will ______________ {converge, diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______________ {converge, diverge} because ____

(Give a reason for your answer such as harmonic series, *p*-series with $p > 1$, *p*-series with $p < 1$, geometric series, *n*th Term Test for Divergence)

The limit $\lim_{n \to \infty} \frac{a_n}{b_n} =$ [ ].

6. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{5n^5 + 8n^2}{\sqrt{n^{12} + 8n^2}}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n =$ [ ] to show $\sum_{n=1}^{\infty} \frac{5n^5 + 8n^2}{\sqrt{n^{12} + 8n^2}}$ will ______________ {converge, diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______________ {converge, diverge} because ____

(Give a reason for your answer such as harmonic series, *p*-series with $p > 1$, *p*-series with $p < 1$, geometric series, *n*th Term Test for Divergence.)

The limit $\lim_{n \to \infty} \frac{a_n}{b_n} =$ [ ].

7. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{7}{\sqrt{n^3 + 4n + 16}}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n =$ [ ] to show $\sum_{n=1}^{\infty} a_n$ will ______________ {converge, diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______________ because ____
{converge, diverge} (Give a reason for your answer such as harmonic series, p-series with $p > 1$, p-series with $p < 1$, geometric series, nth Term Test for Divergence)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{}$.

8. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{8}{2^n - 1}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n = \boxed{}$ to show $\sum_{n=1}^{\infty} a_n$ will ______________
{converge, diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______________ because ____
{converge, diverge} (Give a reason for your answer such as harmonic series, p-series with $p > 1$, p-series with $p < 1$, geometric series, nth Term Test for Divergence)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{}$.

9. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{5(n-1)(n+4)(n+3)(n+2)}{\left(n^3+3n\right)(2n+1)^2}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n = \boxed{}$ to show $\sum_{n=1}^{\infty} a_n$ will ______________
{converge, diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______________ because ____
{converge, diverge} (Give a reason for your answer such as harmonic series, p-series with $p > 1$, p-series with $p < 1$, geometric series, nth Term Test for Divergence)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{}$.

10. Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{9e^n + 4}{2e^{5n} - 2}$. Complete the boxes and blanks.

a. We can use the Limit Comparison Test with $b_n = \boxed{}$ to show $\sum_{n=1}^{\infty} a_n$ will ______________
{converge, diverge}

b. Complete, assuming b_n is what you wrote in the box in part **a**.

$\sum_{n=1}^{\infty} b_n$ will ______________ because ____
{converge, diverge} (Give a reason for your answer such as harmonic series, p-series with $p > 1$, p-series with $p < 1$, geometric series, nth Term Test for Divergence)

The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \boxed{}$.