Practice Questions from HW 27-29

1. Answer the following for the power series $\sum c_n (x-a)^n$. Complete the blanks.

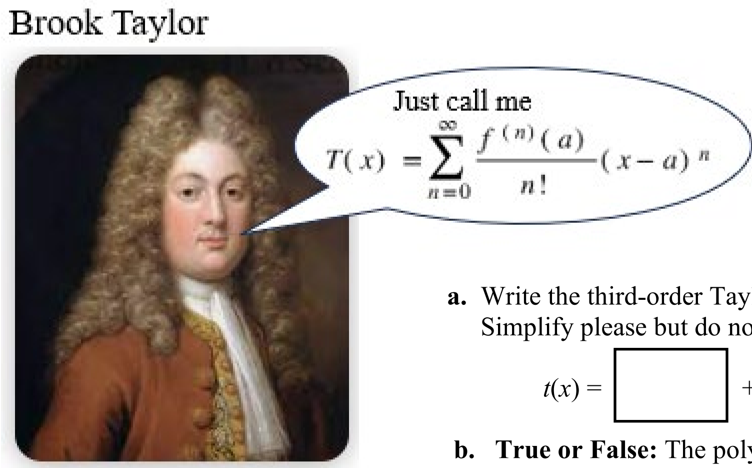
a. The power series $\sum c_n (x-a)^n$ is centered at the value $x =$ ________.

b. Suppose the interval of convergence is **all real numbers**. Then the radius of convergence is $R =$ _______.

c. Suppose the interval of convergence is only **the value** $\boldsymbol{x = a}$. Then the radius of convergence is $R =$ _______.

d. Suppose the interval of convergence is $\boldsymbol{|\, x - a \,| < b}$, i.e. $\boldsymbol{a - b < x < a + b}$.
Then the radius of convergence is $R =$ _______.

2. Suppose we have the following



$f(34) = 60$,

$f'(34) = 43$,

$f''(34) = 22$, and

$f'''(34) = 30$.

a. Write the third-order Taylor polynomial approximation to f at $x = 34$
Simplify please but do not multiply out.

$t(x) =$ $\square$ + $\square$ + $\square$ + $\square$

b. **True or False:** The polynomial $t(x)$ is the tangent cubic to f at the value $x = 34$.

3. A function $f(x)$ centered at 0 has the unique property that $f^{(k)}(0) = 12$ for $k = 0, 1, 2, 3, \ldots$

The Maclaurin Series for $f(x)$ is $\sum_{k=0}^{\infty}$ $\square$ = $\square$ + $\square$ + $\square$ + $\square$ + …

What function $f(x)$ has this property? $f(x) =$ $\square$

For all x we have $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ $\quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

For $-1 < x < 1$ we have $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$

For $-1 < x \le 1$ we have $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ $\quad$ For $-1 \le x \le 1$ we have $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

4. Write the first four nonzero terms of the series for $f(w) = e^{-w}$.
Integrate the series term by term to create the series for $h(w) = \int f(w)\, dw$, assuming $h(0) = -1$.
To what function does the series for $h(w)$ converge?

5. Write the first four nonzero terms of the series for $f(w) = \sin(w^2) =$ _____ + _____ + _____ + ______.
Differentiate the series for $f(w)$ term by term to create the series $g(w) = f'(w) =$ ____ + ____ + ____ + _____.
What function does the series $g(w)$ converge to?

6. Write the first four nonzero terms of the series for $f(w) = \tan^{-1}(w^2) =$ ____ + ____ + ____ + _____.
Report the radius of convergence and the interval of convergence.
Describe what occurs at each endpoint (hole, defined point, vertical asymptote approaching ∞, vertical asymptote approaching $-\infty$).
Indicate what convergence test you used to classify the endpoints. Sketch a graph on the interval of convergence.

7. Write the first four nonzero terms of the series for $f(w) = 10\tan^{-1}(\frac{w}{2}) =$ ____ + ____ + ____ + _____.
Report the radius of convergence and the interval of convergence.
Describe what occurs at each endpoint (hole, defined point, vertical asymptote approaching ∞, vertical asymptote approaching $-\infty$)
Sketch a graph on the interval of convergence.

More follows…

8. Write the first four nonzero terms of the series for $f(x) = e^{-(x-70)^2}$.

9. Consider the Taylor series $-49x^2 + \frac{49x^4}{2} - \frac{49x^8}{3} + \frac{49x^8}{4} - \frac{49x^{10}}{5} + \ldots$

a. Use the set of Fun Facts to write the function that represents this series on its interval of convergence.

b. Write the series using summation notation.

c. Report the radius R of convergence. $R =$ ___

d. Test each endpoint for convergence.

i. When $x = -R$, the series $-49x^2 + \frac{49x^4}{2} - \frac{49x^8}{3} + \frac{49x^8}{4} - \frac{49x^{10}}{5} + \ldots =$ ☐

Write in the box an exact number or DNE or ∞ or $-\infty$.

ii. When $x = R$, the series $-49x^2 + \frac{49x^4}{2} - \frac{49x^8}{3} + \frac{49x^8}{4} - \frac{49x^{10}}{5} + \ldots =$ ☐

Write in the box an exact number or DNE or ∞ or $-\infty$.

e. Report the interval of convergence.

f. Sketch a graph on the interval of convergence.

10. Consider the Taylor series $-5x + \frac{(5x)^2}{2} - \frac{(5x)^3}{3} + \frac{(5x)^4}{4} - \frac{(5x)^5}{5} + \ldots$

a. Use the set of Fun Facts to write the function that represents this series on its interval of convergence.

b. Write the series using summation notation.

c. Report the radius R of convergence. $R =$ ___

d. Test each endpoint for convergence.

i. When $x = -R$, the series $-5x + \frac{(5x)^2}{2} - \frac{(5x)^3}{3} + \frac{(5x)^4}{4} - \frac{(5x)^5}{5} + \ldots =$ ☐

Write in the box an exact number or DNE or ∞ or $-\infty$.

ii. When $x = R$, the series $-5x + \frac{(5x)^2}{2} - \frac{(5x)^3}{3} + \frac{(5x)^4}{4} - \frac{(5x)^5}{5} + \ldots =$ ☐

Write in the box an exact number or DNE or ∞ or $-\infty$.

e. Report the interval of convergence.

f. Sketch a graph on the interval of convergence.

See next page.

Practice Questions over 12.1

1. Eliminate the parameter, t, to obtain an equation of the form $y = f(x)$.

a. $x = \sqrt[3]{t-1}$, $y = \cos t$ (There is no new domain restriction.)

b. $x = \sqrt[3]{t-1}$, $y = \cos \sqrt[3]{t-1}$ (There is no new domain restriction.)

c. $x = -\sqrt{t}$, $y = -3\sqrt{t} + 6e^{\sqrt{t}}$ (Specify the domain restriction.)

d. $x = -\sqrt{t}$, $y = 2t + 1$ (Specify the domain restriction.)

e. $x = e^{-t}$, $y = 7e^{-3t}$ (Specify the domain restriction.)

f. $x = 4\sin t$, $y = 3 + 4\cos t$

g. $x = 3\sin t$, $y = 3 - 6\cos t$

h. $x = 3\cos t$, $y = 3 - 9\cos^2 t$

2. Write a set of parametric equations $x = f(t)$, $y = g(t)$ for the curve

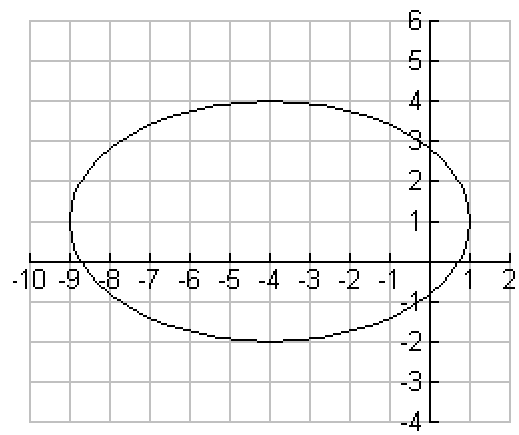
a. $x = 4y^5 - 3y^2 + 2\cos y - e^{7y}$

b. The circle $(x-1)^2 + (y+2)^2 = 49$

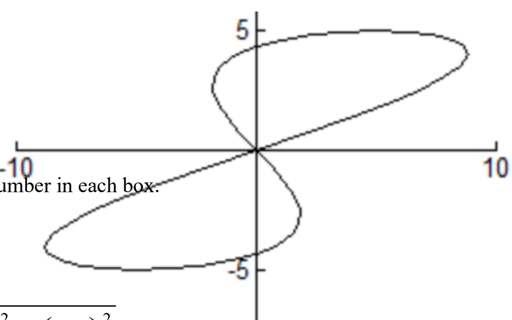
c. The ellipse $\dfrac{(x-5)^2}{9} + \dfrac{(y+2)^2}{4} = 1$

d. The ellipse shown with the initial value of $t = 0$, $x = 1$, $y = 1$ traveling counterclockwise.

e. The same ellipse shown to the right with the initial value of $t = 0$, $x = -4$, $y = -2$ traveling clockwise.



3. The graph of the parametric equations $x = 5\sin t - 5\sin 2t$ and $y = 5\sin t$ is shown for $0 \le t \le 2\pi$.



a. Evaluate $\dfrac{dy}{dx}$ at the origin when $t = 0$. $\dfrac{dy}{dx} = \square$

b. Evaluate $\dfrac{dy}{dx}$ at the origin when $t = \pi$. $\dfrac{dy}{dx} = \square$

Write a number in each box.

c. The arc length from $t = 0$ to $t = 2\pi$ of this curve is given by $\displaystyle\int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$.

Complete the boxes to set up the integral to find the arc length. You need not simplify. Then use FNINT to find the arc length rounded to the nearest whole number.

$$\int_0^{2\pi} \sqrt{\boxed{}}\, dt \approx \boxed{}$$