Practice Questions from HW 25-27

1. Answer the following for the power series of the form $\sum c_n(x-a)^n$. Complete the blanks.

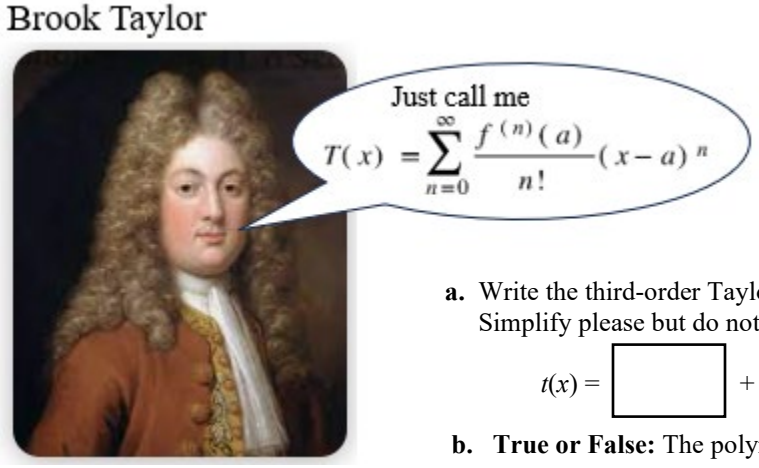
a. The power series $\sum c_n(x-a)^n$ is centered at the value $x =$ _____.

b. Suppose the interval of convergence is **all real numbers**. Example: $\sum x^n / n!$ Then the radius of convergence is $R =$ ____.

c. Suppose the interval of convergence is only **the value $x = 5$**. Example: $\sum n!(x-5)^n$ Then the radius of convergence is $R =$ ____.

d. Suppose the interval of convergence is $|x - 8| < 3$, i.e. $-3 < x - 8 < 3$.
Then the center of the interval of convergence is ____ and the radius of convergence is $R =$ _____.

2. Suppose we have the following



$f(34) = 60$,

$f'(34) = 43$,

$f''(34) = 22$, and

$f'''(34) = 30$.

a. Write the third-order Taylor polynomial approximation to f at $x = 34$
Simplify please but do not multiply out.

$t(x) =$ ☐ + ☐ + ☐ + ☐

b. True or False: The polynomial $t(x)$ is the tangent cubic to f at the value $x = 34$.

3. A function $f(x)$ centered at 0 has the unique property that $f^{(k)}(0) = 12$ for $k = 0, 1, 2, 3, \ldots$

The Maclaurin Series for $f(x)$ is $\sum_{k=0}^{\infty}$ ☐ = ☐ + ☐ + ☐ + ☐ + …

What function $f(x)$ has this property? $f(x) =$ ☐

For all x we have $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

For $-1 < x < 1$ we have $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$

For $-1 < x \leq 1$ we have $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ For $-1 \leq x \leq 1$ we have $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

4. Write the first four nonzero terms of the series for $f(w) = e^{-w}$.
Integrate the series term by term to create the series for $h(w) = \int f(w)\, dw$, assuming $h(0) = -1$.
To what function does the series for $h(w)$ converge?

5. Write the first four nonzero terms of the series for $f(w) = \sin(w^2) =$ _____ + _____ + _____ + _____.
Differentiate the series for $f(w)$ term by term to create the series $g(w) = f'(w) =$ ____ + ____ + ____ + _____.
What function does the series $g(w)$ converge to?

6. Write the first four nonzero terms of the series for $f(w) = \tan^{-1}(w^2) =$ ____ + ____ + ____ + _____.
Report the radius of convergence and the interval of convergence.
Describe what occurs at each endpoint (hole, defined point, vertical asymptote approaching ∞, vertical asymptote approaching $-\infty$).
Indicate what convergence test you used to classify the endpoints. Sketch a graph on the interval of convergence.

7. Write the first four nonzero terms of the series for $f(w) = 10\tan^{-1}(\frac{w}{2}) =$ ____ + ____ + ____ + _____.
Report the radius of convergence and the interval of convergence.
Describe what occurs at each endpoint (hole, defined point, vertical asymptote approaching ∞, vertical asymptote approaching $-\infty$)
Sketch a graph on the interval of convergence.

8. Write the first four nonzero terms of the series for $f(x) = e^{-(x-70)^2}$.

9. Consider the Taylor series $-49x^2 + \frac{49x^4}{2} - \frac{49x^8}{3} + \frac{49x^8}{4} - \frac{49x^{10}}{5} + \ldots$

a. Use the set of Fun Facts to write the function that represents this series on its interval of convergence.

b. Write the series using summation notation.

c. Report the radius R of convergence. $R =$ ___

d. Test each endpoint for convergence.

i. When $x = -R$, the series $-49x^2 + \frac{49x^4}{2} - \frac{49x^8}{3} + \frac{49x^8}{4} - \frac{49x^{10}}{5} + \ldots =$ ☐

Write in the box an exact number or DNE or ∞ or $-\infty$.

ii. When $x = R$, the series $-49x^2 + \frac{49x^4}{2} - \frac{49x^8}{3} + \frac{49x^8}{4} - \frac{49x^{10}}{5} + \ldots =$ ☐

Write in the box an exact number or DNE or ∞ or $-\infty$.

e. Report the interval of convergence.

f. Sketch a graph on the interval of convergence.

10. Consider the Taylor series $-5x + \frac{(5x)^2}{2} - \frac{(5x)^3}{3} + \frac{(5x)^4}{4} - \frac{(5x)^5}{5} + \ldots$

a. Use the set of Fun Facts to write the function that represents this series on its interval of convergence.

b. Write the series using summation notation.

c. Report the radius R of convergence. $R =$ ___

d. Test each endpoint for convergence.

i. When $x = -R$, the series $-5x + \frac{(5x)^2}{2} - \frac{(5x)^3}{3} + \frac{(5x)^4}{4} - \frac{(5x)^5}{5} + \ldots =$ ☐

Write in the box an exact number or DNE or ∞ or $-\infty$.

ii. When $x = R$, the series $-5x + \frac{(5x)^2}{2} - \frac{(5x)^3}{3} + \frac{(5x)^4}{4} - \frac{(5x)^5}{5} + \ldots =$ ☐

Write in the box an exact number or DNE or ∞ or $-\infty$.

e. Report the interval of convergence.

f. Sketch a graph on the interval of convergence.

11. The function $f(x)$ has derivatives of all orders on the interval $-1 < x < 1$ with the following properties.

$f(0) = 0$

$f'(0) = 1$

$f^{(n+1)}(0) = -nf^{(n)}(0)$

a. Find the next five derivatives of f.

$f(0) = 0$

$f'(0) = 1$

$f^{(2)}(0) =$ ☐

$f^{(3)}(0) =$ ☐

$f^{(4)}(0) =$ ☐

$f^{(5)}(0) =$ ☐

$f^{(6)}(0) =$ ☐

b. The Maclaurin series converges for all $|x| < 1$. Write the first six nonzero terms of f.

c. Write the function $f(x)$ in sigma notation.

d. Write the first six nonzero terms of the function $h(x) = f'(x)$.

e. Write $h(x)$ in sigma notation.

f. Knowing what you do about the series $h(x)$, construct (non-series) formulas for h and f.

12. The function f has a Taylor series about $x = 1$ that converges to $f(x)$ for all x in the interval of convergence. It is known that

$f(1) = 1$

$f'(1) = -\frac{1}{2}$

$f^{(n)}(1) = -\frac{(n-1)!}{2^n}$ for all $n \geq 2$

a. Evaluate the first five derivatives of f at $x = 1$.

b. Write the first six nonzero terms of the Taylor series for f about $x = 1$.

c. Write the function $f(x)$ in sigma notation. $f(x) = \sum_{n=1}^{\infty} \frac{-(x-1)^n}{n \cdot 2^n}$

d. The Taylor series for f about $x = 1$ has a radius of convergence of 2. Find the interval of convergence by testing each endpoint. Show work.

Practice Questions over 12.1

1. Eliminate the parameter, t, to obtain an equation of the form $y = f(x)$.

a. $x = \sqrt[3]{t-1}$, $y = \cos t$ (There is no new domain restriction.)

b. $x = \sqrt[3]{t-1}$, $y = \cos \sqrt[3]{t-1}$ (There is no new domain restriction.)

c. $x = -\sqrt{t}$, $y = -3\sqrt{t} + 6e^{\sqrt{t}}$ (Specify the domain restriction.)

d. $x = -\sqrt{t}$, $y = 2t + 1$ (Specify the domain restriction.)

e. $x = e^{-t}$, $y = 7e^{-3t}$ (Specify the domain restriction.)

f. $x = 4\sin t$, $y = 3 + 4\cos t$

g. $x = 3\sin t$, $y = 3 - 6\cos t$



Solve for cost t and sin t and use the Pythagorean Identity $\cos^2 t + \sin^2 t = 1$

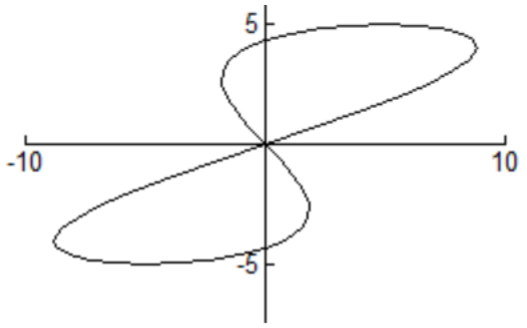
h. $x = 3\cos t$, $y = 3 - 9\cos^2 t$ (Specify the domain restriction.)

2. The graph of the parametric equations $x = 5\sin t - 5\sin 2t$
and $y = 5\sin t$
is shown for $0 \le t \le 2\pi$.

a. Evaluate $\frac{dy}{dx}$ at the origin when $t = 0$. $\frac{dy}{dx} =$ ☐

b. Evaluate $\frac{dy}{dx}$ at the origin when $t = \pi$. $\frac{dy}{dx} =$ ☐

Insert numbers in each box.



c. Determine the values of t, x, and y on $0 \le t < 2\pi$ when the curve has a horizontal tangent.

d. The arc length from $t = 0$ to $t = 2\pi$ of this curve is given by $\int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$.

Complete the boxes to set up the integral to find the arc length. You need not simplify.
Then use FNINT to find the arc length rounded to the nearest whole number.

$$\int_0^{2\pi} \sqrt{\square}\, dt \approx \square$$