Practice Questions from 11.1-11.4

1. Answer the following for the power series $\sum c_n (x-a)^n$. Complete the blanks.

a. The power series $\sum c_n (x-a)^n$ is centered at the value $x =$ ________.

b. Suppose the interval of convergence is **all real numbers**. Then the radius of convergence is $R =$ _______.

c. Suppose the interval of convergence is only **the value $x = a$**. Then the radius of convergence is $R =$ _______.

d. Suppose the interval of convergence is $|\,x - a\,| < b$, i.e. $a - b < x < a + b$.

Then the radius of convergence is $R =$ _______.

2. The interval of convergence of $\sum_{n=1}^{\infty} \left(\frac{x-4}{2} \right)^n$ is $\square < x < \square$. Show work below.

Hint: It is a geometric series.

3. Suppose we have the following

$f(34) = 60$,

$f'(34) = 43$,

$f''(34) = 22$, and

$f'''(34) = 30$.

a. Write the third-order Taylor polynomial approximation to f at $x = 34$ in expanded form. Simplify please.

$t(x) =$

b. **True or False:** The polynomial $t(x)$ is the tangent cubic to f at the value $x = 34$. _________

4. Report the interval of convergence of $\sum_{n=0}^{\infty} n!x^{5n}$. Select one.

A. $-1<x<1$ B. $-\frac{1}{\sqrt{5}}<x<\frac{1}{\sqrt{5}}$ C. $-\sqrt[5]{5}<x<\sqrt[5]{5}$ D. $x=0$ E. $-\frac{1}{5}<x<\frac{1}{5}$ F. $-\infty<x<\infty$

5. The interval of convergence of $\sum_{n=1}^{\infty} \frac{x^{3n}}{n!}$ is $\square < x < \square$. Show work below.

6. Consider $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$

a. The radius of convergence is $R =$ ______. Show work below.

b. If x is equal to the **left endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$ will ________________.
{converge, diverge}

c. If x is equal to the **right endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$ will________________.
{converge, diverge}

d. State the **reasons** which justify your claims about the endpoints in parts **b** and **c**.

7. Consider $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$

a. The radius of convergence is $R =$ ______. Show work below.

b. If x is equal to the **left endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$ will ________________.
{converge, diverge}

c. If x is equal to the **right endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$ will________________.
{converge, diverge}

d. State the **reasons** which justify your claims about the endpoints in parts **b** and **c**.

Fun Facts:

For all x we have $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ $\quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

For $-1 < x < 1$ we have $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$

For $-1 < x \le 1$ we have $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ $\quad$ For $-1 \le x \le 1$ we have $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

8. Complete: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots =$ ☐. The name of this series is called the ____________________ series.

TIP: Use a Fun Fact above.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

9. Complete: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots =$ ☐. The name of this series is called the ____________________ series.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

10. a. In sigma notation the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots = \sum_{\square}^{\infty} \square$

b. Use one of the Fun Facts above to determine what function $f(x)$ the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots$ approximates.

$f(x) =$ ☐. .The radius of convergence is $R =$ ☐

Simplified please.

c. Write the first four terms of the series in expanded form if $x = -1$. ☐ $- \ldots$

The left endpoint $x = -1$ ________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: ____

Simplified please.

d. Write the first four terms of the series in expanded form if $x = 1$. ☐ $- \ldots$

The right endpoint $x = 1$ ________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: ____

11. Write the first four nonzero terms of the series for $f(w) = 10\tan^{-1}(2w)$.

12. Write the first four nonzero terms of the series for $f(w) = \sin(w^2)$.

13. Write the first four nonzero terms of the series for $f(w) = e^{-w}$.

14. The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ is the power series below.

a. Write the first four nonzero terms of the series for $f'(x)$.

Simplified please

$f'(x) =$ [ ] $+ \ldots$

b. The radius of convergence of $f'(x)$ is $R =$ ____

c. If x is equal to the **left endpoint** of the interval of convergence, the series for $f'(x)$ will __________________.
{converge, diverge}

d. If x is equal to the **right endpoint** of the interval of convergence, the series for $f'(x)$ will __________________.
{converge, diverge}

e. Write the series for $f'(x)$ in sigma notation.

$f'(x) = \sum_{n = \square}^{\infty} \left(\square \right)$

f. When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?

$f'(x) = \dfrac{\square}{\square}$

g. Sketch a graph of $\sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ on its the interval of convergence.