## Practice Questions from 10.7-10.8 and 11.1-11.2

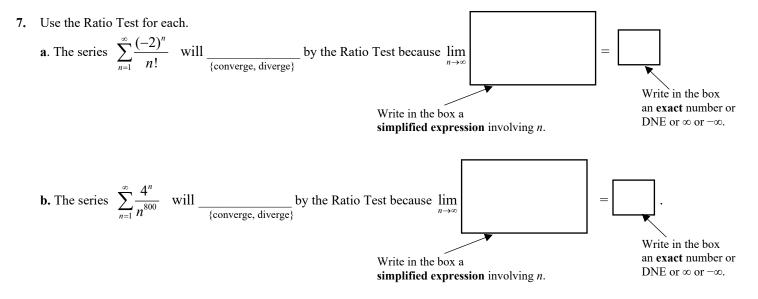
- The Ratio Test and Root Test are based on the properties of convergence of
   A. a p-series, p≠1.
   B. the harmonic series
   C. the alternating series
   D. a television series
   E. the world series. F. a geometric series
- 2. Which of these will help you determine if the series  $\sum_{n=0}^{\infty} 2e^n$  converges or diverges? Select all possible answers.

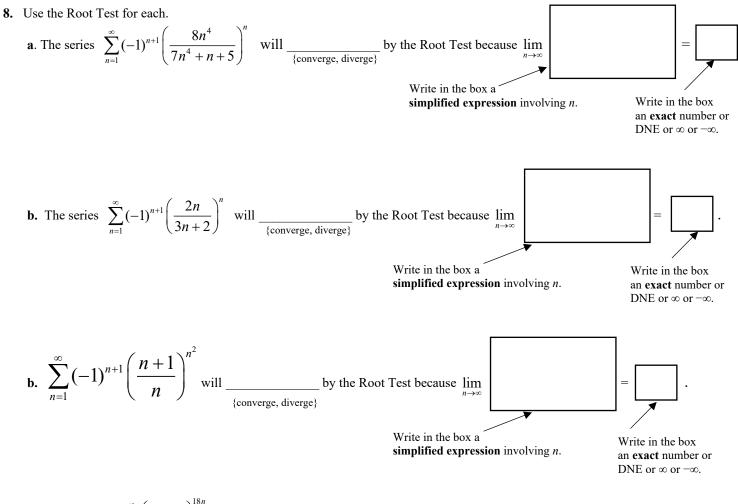
A. limit comparison test with a *p*-series,  $p \neq 1$ . B. limit comparison test with the harmonic series C. a geometric series

C. alternating series test E. absolute convergence test (i.e., convergence of  $\sum |a_n|$  implies convergence of  $\sum a_n$ )

D. integral test E. ratio test F. nth Term Test for Divergence

- Which of these will help you determine if the series ∑<sub>n=0</sub><sup>∞</sup> e<sup>-2n</sup> converges or diverges? Select all possible answers.
   A. limit comparison test with a *p*-series, *p*≠1. B. limit comparison test with the harmonic series C. a geometric series C. alternating series test E. absolute convergence test (i.e., convergence of ∑|a<sub>n</sub>| implies convergence of ∑a<sub>n</sub>)
   D. integral test E. ratio test F. *n*th Term Test for Divergence
- 4. Which of these will help you determine if the series  $\sum_{n=1}^{\infty} \left( \frac{(-1)^{n+1}}{n^2} \right)$  converges or diverges? Select all possible answers. A. limit comparison test with a *p*-series,  $p \neq 1$ . B. limit comparison test with the harmonic series C. a geometric series C. alternating series test E. absolute convergence test (i.e., convergence of  $\sum |a_n|$  implies convergence of  $\sum a_n$ ) D. ratio test E. *n*th Term Test for Divergence
- 5. Which of these will help you determine if the series  $\sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{\sqrt{n}}\right)$  converges or diverges? Select all possible answers. A. limit comparison test with a *p*-series,  $p \neq 1$ . B. limit comparison test with the harmonic series C. a geometric series C. alternating series test E. absolute convergence test (i.e., convergence of  $\sum |a_n|$  implies convergence of  $\sum a_n$ ) D. ratio test E. *n*th Term Test for Divergence
- 6. Which of these will help you determine if the series  $\sum_{n=1}^{\infty} \left(\frac{n+2}{n!}\right)$  converges or diverges? Select all possible answers. A. limit comparison test with a *p*-series,  $p \neq 1$ . B. limit comparison test with the harmonic series C. a geometric series C. alternating series test E. absolute convergence test (i.e., convergence of  $\sum |a_n|$  implies convergence of  $\sum a_n$ ) D. integral test E. ratio test F. *n*th Term Test for Divergence





9. Consider the series 
$$\sum_{n=1}^{\infty} \left(1 + \frac{a}{n}\right)^{15n}$$
 for some real number **a**.

**b.** Circle the best answer to determine part **a**.

A. It is a p-series. B. It is a geometric series C. Use the Ratio Test D. Use the Root Test E. Use the nth Term Test for Divergence

c. Explain more fully below how part b justifies part a.

10. Answer the following for the power series  $\sum c_n (x-a)^n$ . Complete the blanks.

- **a.** The power series  $\sum c_n (x-a)^n$  is centered at the value x =\_\_\_\_\_.
- **b.** Suppose the interval of convergence is **all real numbers**. Then the radius of convergence is R =\_\_\_\_\_.
- **c.** Suppose the interval of convergence is only the value x = a. Then the radius of convergence is R =\_\_\_\_\_
- **d.** Suppose the interval of convergence is |x a| < b, i.e. a b < x < a + b. Then the radius of convergence is R =\_\_\_\_\_.

11. The interval of convergence of 
$$\sum_{n=1}^{\infty} \left(\frac{x-4}{2}\right)^n$$
 is  $|| < x < ||$ . Show work below.

Hint: It is a geometric series.

12. Report the interval of convergence of  $\sum_{n=0}^{\infty} n! x^{5n}$ . Select one.

A. 
$$-1 < x < 1$$
 B.  $-\frac{1}{\sqrt{5}} < x < \frac{1}{\sqrt{5}}$  C.  $-\sqrt[5]{5} < x < \sqrt[5]{5}$  D.  $x = 0$  E.  $-\frac{1}{5} < x < \frac{1}{5}$  F.  $-\infty < x < \infty$ 

**13.** The interval of convergence of  $\sum_{n=1}^{\infty} \frac{x^{3n}}{n!}$  is || < x < ||. Show work below.

14. Consider  $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$ 

**a**. The radius of convergence is R =\_\_\_\_\_. Show work below.

- **b.** If x is equal to the **left endpoint** of the interval of convergence, the series  $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$  will  $\frac{1}{\{\text{converge, diverge}\}}$ .
- d. State the reasons which justify your claims about the endpoints in parts b and c.
- 15. Consider  $\sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{x^{n+11}}{n^2} \right)$ 
  - **a**. The radius of convergence is R =\_\_\_\_\_. Show work below.

**b.** If x is equal to the **left endpoint** of the interval of convergence, the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{x^{n+11}}{n^2} \right)$  will \_\_\_\_\_\_\_.

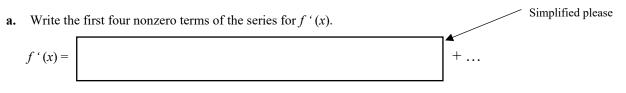
c. If x is equal to the **right endpoint** of the interval of convergence, the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{x^{n+11}}{n^2} \right) \text{will}_{\{\text{converge, diverge}\}}.$ 

d. State the reasons which justify your claims about the endpoints in parts b and c.

Fun Facts:

For all x we have 
$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{2}}{3!} + \frac{x^{4}}{4!} + \cdots$$
 sin  $x = x - \frac{x^{3}}{3!} + \frac{x^{5}}{5!} - \frac{x^{2}}{7!} + \cdots$  cos  $x = 1 - \frac{x^{2}}{2!} + \frac{x^{4}}{4!} - \frac{x^{4}}{6!} + \cdots$   
For  $-1 < x < 1$  we have  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^{n} = 1 + x + x^{2} + x^{3} + x^{4} + \cdots$   
For  $-1 < x < 1$  we have  $\ln(1+x) = x - \frac{x^{2}}{2} + \frac{x^{2}}{3} - \frac{x^{4}}{4} + \cdots$   
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For  $-1 \le x \le 1$  we have  $\tan^{-1}x = x - \frac{x^{2}}{3} + \frac{x^{2}}{5} - \frac{x^{2}}{7} + \cdots$   
16. Complete:  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots =$ 
The name of this series is called the series.  
TP: Use a Fun Fact above.  
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The name of this series  $x = x - \frac{x^{2}}{2} - \frac{x^{2}}{3} - \frac{x^{4}}{4} - \frac{x^{2}}{5} - \cdots$  approximates.  
 $f(x) =$ 
The radius of convergence is  $R =$ 
The name of the series  $x = x - \frac{x^{2}}{2} - \frac{x^{2}}{3} - \frac{x^{4}}{4} - \frac{x^{2}}{5} - \cdots$  approximates.  
 $f(x) =$ 
The radius of convergence is  $R =$ 
The name of the series in expanded form if  $x = -1$ .  
The right endpoint  $x = -1$  in the interval of convergence. Explain your answer.  
Reason:  
The right endpoint  $x = 1$  in the interval of convergence. Explain your answer.  
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The right endpoint  $x = 1$  in the interval of convergence. Expla

22. The term-by-term derivative of  $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$  is the power series below.



- **b.** The radius of convergence of f'(x) is R =\_\_\_\_\_
- c.

- If x is equal to the **right endpoint** of the interval of convergence, the series for f'(x) will\_ d. {converge, diverge}
- Write the series for f'(x) in sigma notation. e.

$$f'(x) = \sum_{n=1}^{\infty} \left( \boxed{ \boxed{ } \end{aligned} \right)$$

When x is in the interval of convergence, we can write the series for f'(x) as what rational function? f.

