Practice Questions from 10.7-10.8 and 11.1-11.2

1. The Ratio Test and Root Test are based on the properties of convergence of
A. a p-series, $p\neq 1$. B. the harmonic series C. the alternating series D. a television series E. the world series. F. a geometric series

2. Which of these will help you determine if the series $\sum_{n=0}^{\infty} 2e^n$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
C. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
D. integral test E. ratio test F. nth Term Test for Divergence

3. Which of these will help you determine if the series $\sum_{n=0}^{\infty} e^{-2n}$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
C. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
D. integral test E. ratio test F. nth Term Test for Divergence

4. Which of these will help you determine if the series $\sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{n^2} \right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
C. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
D. ratio test E. nth Term Test for Divergence

5. Which of these will help you determine if the series $\sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{\sqrt{n}} \right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
C. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
D. ratio test E. nth Term Test for Divergence

6. Which of these will help you determine if the series $\sum_{n=1}^{\infty} \left(\frac{n+2}{n!} \right)$ converges or diverges? Select all possible answers.
A. limit comparison test with a p-series, $p\neq 1$. B. limit comparison test with the harmonic series C. a geometric series
C. alternating series test E. absolute convergence test (i.e., convergence of $\sum |a_n|$ implies convergence of $\sum a_n$)
D. integral test E. ratio test F. nth Term Test for Divergence

7. Use the Ratio Test for each.

a. The series $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$ will ______________ {converge, diverge} by the Ratio Test because $\lim_{n\to\infty}$ [ ] = [ ]

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

b. The series $\sum_{n=1}^{\infty} \frac{4^n}{n^{800}}$ will ______________ {converge, diverge} by the Ratio Test because $\lim_{n\to\infty}$ [ ] = [ ] .

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

8. Use the Root Test for each.

a. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\dfrac{8n^4}{7n^4+n+5}\right)^n$ will ______________ by the Root Test because $\lim_{n\to\infty}$ [ ] = [ ]
{converge, diverge}

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

b. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\dfrac{2n}{3n+2}\right)^n$ will ______________ by the Root Test because $\lim_{n\to\infty}$ [ ] = [ ] .
{converge, diverge}

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

b. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\dfrac{n+1}{n}\right)^{n^2}$ will ______________ by the Root Test because $\lim_{n\to\infty}$ [ ] = [ ] .
{converge, diverge}

Write in the box a **simplified expression** involving n.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

9. Consider the series $\sum_{n=1}^{\infty}\left(1+\dfrac{a}{n}\right)^{18n}$ for some real number **a**.

a. The series will ________________ .
{converge, diverge}

b. Circle the best answer to determine part **a**.
A. It is a *p*-series. B. It is a geometric series C. Use the Ratio Test D. Use the Root Test E. Use the *n*th Term Test for Divergence

c. Explain more fully below how part **b** justifies part **a**.

10. Answer the following for the power series $\sum c_n(x-a)^n$. Complete the blanks.

a. The power series $\sum c_n(x-a)^n$ is centered at the value $x =$ ________.

b. Suppose the interval of convergence is **all real numbers**. Then the radius of convergence is $R =$ _______.

c. Suppose the interval of convergence is only **the value** $\boldsymbol{x = a}$. Then the radius of convergence is $R =$ _______.

d. Suppose the interval of convergence is $\boldsymbol{|x-a| < b}$, i.e. $\boldsymbol{a-b < x < a+b}$. Then the radius of convergence is $R =$ _______.

11. The interval of convergence of $\sum_{n=1}^{\infty}\left(\dfrac{x-4}{2}\right)^n$ is [ ] $< x <$ [ ] . Show work below.

Hint: It is a geometric series.

12. Report the interval of convergence of $\sum_{n=0}^{\infty} n!x^{5n}$. Select one.

A. $-1<x<1$ B. $-\frac{1}{\sqrt{5}}<x<\frac{1}{\sqrt{5}}$ C. $-\sqrt[5]{5}<x<\sqrt[5]{5}$ D. $x=0$ E. $-\frac{1}{5}<x<\frac{1}{5}$ F. $-\infty<x<\infty$

13. The interval of convergence of $\sum_{n=1}^{\infty} \frac{x^{3n}}{n!}$ is $\square$ $< x <$ $\square$. Show work below.

14. Consider $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$

a. The radius of convergence is $R =$ ______. Show work below.

b. If x is equal to the **left endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$ will ________________.
{converge, diverge}

c. If x is equal to the **right endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty} \frac{(3x)^n}{n}$ will________________.
{converge, diverge}

d. State the **reasons** which justify your claims about the endpoints in parts **b** and **c**.

15. Consider $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$

a. The radius of convergence is $R =$ ______. Show work below.

b. If x is equal to the **left endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$ will ________________.
{converge, diverge}

c. If x is equal to the **right endpoint** of the interval of convergence, the series $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{x^{n+11}}{n^2}\right)$ will________________.
{converge, diverge}

d. State the **reasons** which justify your claims about the endpoints in parts **b** and **c**.

Fun Facts:

For all x we have $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

For $-1 < x < 1$ we have $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$

For $-1 < x \le 1$ we have $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ For $-1 \le x \le 1$ we have $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

16. Complete: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots =$ ☐ . The name of this series is called the ____________________ series.

TIP: Use a Fun Fact above.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

17. Complete: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots =$ ☐ . The name of this series is called the ____________________ series.

Write in the box an **exact** number or DNE or ∞ or $-\infty$.

Be specific please.

18. a. In sigma notation the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots = \sum^{\infty}$ ☐

b. Use one of the Fun Facts above to determine what function $f(x)$ the series $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots$ approximates.

$f(x) =$ ☐ .The radius of convergence is $R =$ ☐

Simplified please.

c. Write the first four terms of the series in expanded form if $x = -1$. ☐ $+ \ldots$

The left endpoint $x = -1$ ________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: ____

Simplified please.

d. Write the first four terms of the series in expanded form if $x = 1$. ☐ $- \ldots$

The right endpoint $x = 1$ ________ in the interval of convergence. Explain your answer.
{is, is not}

Reason: ____

19. Consider function $f(x) = 10\tan^{-1}(2w)$. Write the first **four** terms of the series.

Simplified please.

$10\tan^{-1}(2w) =$ ☐ $+ \ldots$

20. Consider function $f(w) = \sin(w^2)$. Write the first **four** terms of the series.

Simplified please.

$\sin(w^2) =$ ☐ $+ \ldots$

21. Consider function $f(w) = e^{-w}$. Write the first **four** terms of the series.

Simplified please.

$e^{-w} =$ ☐ $+ \ldots$

22. The term-by-term derivative of $f(x) = \sum_{n=0}^{\infty} 5x^n = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + \cdots$ is the power series below.

a. Write the first four nonzero terms of the series for $f'(x)$.

Simplified please

$f'(x) =$ ☐ $+ \ldots$

b. The radius of convergence of $f'(x)$ is $R =$ ____

c. If x is equal to the **left endpoint** of the interval of convergence, the series for $f'(x)$ will ________________.
{converge, diverge}

d. If x is equal to the **right endpoint** of the interval of convergence, the series for $f'(x)$ will________________.
{converge, diverge}

e. Write the series for $f'(x)$ in sigma notation.

$f'(x) = \sum_{n = \square}^{\infty} \left(\square \right)$

f. When x is in the interval of convergence, we can write the series for $f'(x)$ as what rational function?

$f'(x) = \dfrac{\square}{\square}$