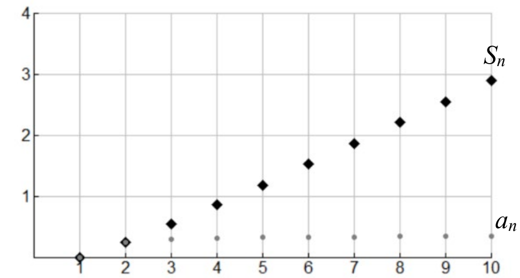
Practice Questions from Section 10.4

1. For what values of p does $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converge? ____________ For what values of r does $\sum_{k=1}^{\infty} ar^{k-1}$ converge? ____________

2. Shown to the right is a plot of the terms a_n and the nth partial sums S_n for the series $\sum_{k=1}^{\infty}\left(1-\frac{1}{k}\right)^k$.



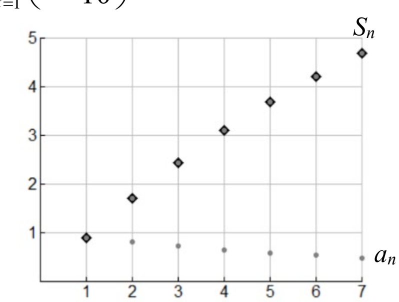
a. Circle the correct choice: We know that $\lim_{k\to\infty}\left(1-\frac{1}{k}\right)^k$ __________ equal 0.
{does, does not}

This means by the nth Term Test for Divergence that the series $\sum_{k=1}^{\infty}\left(1-\frac{1}{k}\right)^k$ ______________________.
{converges, diverges, is nonconclusive}

b. The plot shows $\lim_{k\to\infty}\left(1-\frac{1}{k}\right)^k \approx 0.3678794411714423215955237701614608674458111310317678345078368016974614957448998034$

What is the exact value of this number? $\lim_{k\to\infty}\left(1-\frac{1}{k}\right)^k = \boxed{}$ Write in the box an **exact** number or DNE or ∞ or $-\infty$.

3. Shown to the right is a plot of the terms a_n and the nth partial sums S_n for the series $\sum_{k=1}^{\infty}\left(1-\frac{1}{10}\right)^k$.



a. Circle the correct choice: We know that $\lim_{k\to\infty}\left(1-\frac{1}{10}\right)^k$ __________ equal 0,
{does, does not}

This means by the nth Term Test for Divergence that the series $\sum_{k=1}^{\infty}\left(1-\frac{1}{10}\right)^k$ ______________________.
{converges, diverges, is nonconclusive}

b. The series $\sum_{k=1}^{\infty}\left(1-\frac{1}{10}\right)^k = \boxed{}$ Write in the box an **exact** number or DNE or ∞ or $-\infty$.

4. Suppose $\sum_{k=1}^{\infty} a_k$ is any series. Circle True or False. If False, give a counterexample which shows it is False. If True, leave blank.

a. True False If $\lim_{k\to\infty} a_k = 0$, then $\sum_{k=1}^{\infty} a_k$ converges. ______________________________

b. True False If the limit of the sequence of partial sums S_n exists, i.e., $\lim_{n\to\infty} S_n = \lim_{n\to\infty}\sum_{k=1}^{n} a_k = L < \infty$, then $\sum_{k=1}^{\infty} a_k = L$, where L is some finite number.

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c. True False If the limit of the sequence a_k exists, i.e. $\lim_{k\to\infty} a_k = L$, then the series $\sum_{k=1}^{\infty} a_k$ converges.

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d. True False If $\sum_{k=1}^{\infty} a_k$ converges, then the sequence of partial sums S_n approaches 0, i.e, $\lim_{n\to\infty} S_n = \lim_{n\to\infty}\sum_{k=1}^{n} a_k = 0$.

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5. Determine whether $\sum_{n=1}^{\infty} \frac{560n^4 \cdot 4^n + 11^n}{n^4 11^n}$ converges or diverges by answering the questions below.

a. Circle the correct answer below.

A. The series is the sum of a geometric series with $|r| < 1$ and the harmonic series.
B. The series is the sum of a geometric series with $|r| < 1$ and a p-series with $p < 1$.
C. The series is the sum of a geometric series with $|r| < 1$ and a p-series with $p > 1$.
D. The series is the sum of a geometric series with $|r| > 1$ and the harmonic series.
E. The series is the sum of a geometric series with $|r| > 1$ and a p-series with $p < 1$.
F. The series is the sum of a geometric series with $|r| > 1$ and a p-series with $p > 1$.

b. Write $\sum_{n=1}^{\infty} \frac{560n^4 \cdot 4^n + 11^n}{n^4 11^n}$ as the sum of a geometric series and a p-series (or harmonic series).

c. Circle the correct choice and fill in the blanks: The geometric series has $r =$ ____ and will ________________ {converge, diverge}

and the p-series (or harmonic series) has $p =$ _____ and will ________________ {converge, diverge}.

Thus the series $\sum_{n=1}^{\infty} \frac{560n^4 \cdot 4^n + 11^n}{n^4 11^n}$ will ________________ {converge, diverge}.

6. Because $f(x) = \frac{24}{1+x^2}$ is ______________ {increasing, decreasing} for $x \geq 1$ we can use the Integral Test to show $\sum_{n=1}^{\infty} \frac{24}{1+n^2}$ converges or diverges.

a. The series will ______________ {converge, diverge} because $\int_1^{\infty} \frac{24}{1+x^2} dx = \boxed{}$. Write in the box an **exact** number or DNE or ∞ or −∞.

b. Show work below using correct limit notation:

$\int_1^{\infty} \frac{24}{1+x^2} dx =$

c. Shown below is a plot of a_n and S_n for the series $\sum_{n=1}^{\infty} \frac{24}{1+n^2}$, as well as the area representing $\int_1^{\infty} \frac{24}{1+x^2} dx$.

Circle the best choice:

From the graphs we expect that $\sum_{n=1}^{\infty} \frac{24}{1+n^2}$ ________ $\{=, <, >\}$ $\int_1^{\infty} \frac{24}{1+x^2} dx$