Practice Questions from Section 10.1 – 10.3

1. **a.** For what values of r does the sequence $\{r^n\}$ converge? ________________________

b. For what values of r does the sequence $\{r^n\}$ diverge? ________________________

2. Complete: $\lim_{n\to\infty} (-1)^{n+1}(2)^n =$ [ ] Write in the box an exact number or DNE or ∞ or −∞.

3. Complete: $\lim_{n\to\infty} \left(1 + \frac{\ln\sqrt{7}}{n}\right)^n =$ [ ] Write in the box an exact number or DNE or ∞ or −∞.

4. Consider the sequence given by $\{a_n\}_{n=1}^{\infty}$: $\frac{7}{32}, \frac{7^2}{34}, \frac{7^3}{36}, \frac{7^4}{38}, \frac{7^5}{40}, \ldots$

Find a formula for the nth term of the sequence in terms of n. $\{a_n\}_{n=1}^{\infty} =$ [ ]

5. **a.** Write the next two terms of the sequence $\{a_n\}_{n=1}^{\infty}$: $\pi, \ -\pi, \ \pi, \ -\pi,$ ______, ______

b. Find a formula for the nth term of the sequence in terms of n. $\{a_n\}_{n=1}^{\infty} =$ [ ]

c. Write a recurrence relation that generates the above sequence.

$a_{n+1} =$ [ ] , $a_1 =$ [ ] , for $n = 1, 2, 3, \ldots$.

6. Consider the sequence given by the recurrence relation $a_{n+1} = 2 + \frac{3}{a_n}, \quad a_1 = 1$

a. The sequence converges to a limit L. Give the exact value of L. $L =$ [ ]

b. Convergence occurs when $a_{n+1} = a_n$. Use this fact to write an equation that involves L.

Equation: ____

c. Solve the equation in part **b** to justify your claim in part **a**. You will get two solutions. Reject the negative value of L.

7. Complete: $\sum_{k=0}^{\infty} 400(1.10)^k =$ [ ] Write in the box an exact number or DNE or ∞ or −∞.

8. Complete: $\sum_{k=0}^{\infty} \frac{282}{13^{k-1}} =$ [ ] Write in the box an exact number or DNE or ∞ or −∞.

If $\sum_{k=0}^{\infty} \frac{282}{13^{k-1}}$ were written as $\sum_{k=0}^{\infty} ar^k$, report a and r. $a =$ _______, $r =$ _________.

9. The series $\sum_{k=0}^{\infty} ar^k$ converges to 5. If $a = 9.5$, what is the value of r? Complete: $\sum_{k=0}^{\infty} 9.5\left(\boxed{}\right)^k = 5$

Show work.

10. The series $\sum_{k=0}^{\infty} ar^k$ converges to 5. If $r = \frac{1}{25}$, what is the value of a? Complete: $\sum_{k=0}^{\infty} \boxed{}\left(\frac{1}{25}\right)^k = 5$

Show work.

11. Consider the sequence $\sqrt{42}$, $\sqrt{42-\sqrt{42}}$, $\sqrt{42-\sqrt{42-\sqrt{42}}}$, $\sqrt{42-\sqrt{42-\sqrt{42-\sqrt{42}}}}$, ….

a. Write a recurrence relation that generates the above sequence.

$a_{n+1} = \boxed{}$, $a_1 = \boxed{}$, for $n = 1, 2, 3, \ldots$.

b. The sequence converges to a limit L. Give the exact value of L. $L = \boxed{}$

c. Convergence occurs when $a_{n+1} = a_n$. Use this fact to rewrite the above recurrence relation into an equation that involves L.

Equation: ____

d. ***Solve*** the equation part **c** to justify your claim in part **b**.

12. For what values of r does the series $\sum_{k=0}^{\infty} a(r)^k$ converge? ________________________

13. Consider the function $f(x) = \sum_{k=0}^{\infty} 9\left(\frac{x-4}{2}\right)^k$

a. Evaluate $f(3)$. Show work. $f(3) = \boxed{}$ Write in the box an exact number or DNE or ∞ or −∞.

b. For what values of x does $f(x)$ converge? Show work.

$\boxed{} < x < \boxed{}$

14. Complete: $\frac{2\cdot 124}{125} + \frac{2\cdot 124^2}{125^2} + \frac{2\cdot 124^3}{125^3} + \ldots = \boxed{}$ Write in the box an exact number or DNE or ∞ or −∞.

15. Consider the sequence given by the recurrence relation $a_{n+1} = 0.95a_n + 8.2, \quad a_1 = 8.2$

a. The sequence converges to a limit L. Give the exact value of L. $L = \square$

b. Convergence occurs when $a_{n+1} = a_n$. Use this fact to rewrite the above recurrence relation into an equation that involves L.

Equation: ______________________________________

c. ***Solve*** the equation in part **b** to justify your claim in part **a**.

d. Complete the boxes below to write the next two terms of the series in long form.
Each subsequent term involves a numerical expression containing 0.95 and 8.2.

$8.2 + \square + \square + \ldots$

e. Without using sigma notation, write an expression that gives the nth partial sum of this series $S_n = \dfrac{\square\left(1 - \square^{\square}\right)}{\square}$
i.e., the sum of the series of n terms.

f. Enter your expression from part **e** in your grapher and scroll a table to find the value of n for which the sum first surpasses 150.

The number of terms $n = \square$

16. Once per year Richie Rich deposits an amount of $400 in an account which pays 10% interest per year, compounded annually, with **<u>additional deposits of $400 continually made at the end of the year</u>**.

If B_n is the balance in the account, in dollars, immediately after Richie makes the nth deposit, then we can write $B_1 = \$400$.

a. Complete the table to find the following. Report to the nearest $0.01.

i) the balance, B_2, of the account on the day immediately after the second deposit.
ii) the balance, B_3, of the account on the day immediately after the third deposit.
iii) the balance, B_4, of the account on the day immediately after the fourth deposit.

n, # Deposits	B_n
1	$400
2	
3	
4	

b. Suppose Richie makes 422 deposits. Which is true about the sum B_{422}?
The balance, B_{422}, of the account on the day immediately after the 422nd deposit is exactly

A. $B_{422} = 400\cdot10^{422} + 400\cdot10^{421} + \ldots + 400\cdot10^2 + 400\cdot10 + 400$

B. $B_{422} = 400\cdot1.10^{423} + 400\cdot1.10^{422} + \ldots + 400\cdot1.10^2 + 400\cdot1.10 + 400$

C. $B_{422} = 400\cdot10^{423} + 400\cdot10^{422} + \ldots + 400\cdot10^2 + 400\cdot10 + 400$

D. $B_{422} = 400\cdot1.10^{422} + 400\cdot1.10^{421} + \ldots + 400\cdot1.10^2 + 400\cdot1.10 + 400$

E. $B_{422} = 400\cdot1.10^{421} + 400\cdot1.10^{420} + \ldots + 400\cdot1.10^2 + 400\cdot1.10 + 400$

F. $B_{422} = 400\cdot10^{421} + 400\cdot10^{420} + \ldots + 400\cdot10^2 + 400\cdot10 + 400$

c. The balance, B_{422}, of the account on the day immediately after the 422nd deposit is approximately

A. $B_{422} \approx \$1291712354137103000000$

B. $B_{422} \approx \$1067530871187688000000$

C. $B_{422} \approx \$1174283958306457000000$

D. $B_{422} \approx \$1188774622351958700000$

E. $B_{422} \approx \$14490664045501680000$

F. The value of B_{422} can not be computed.