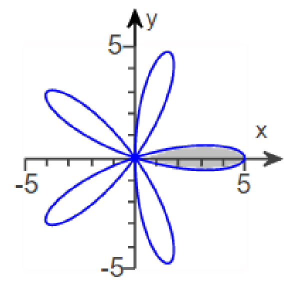
MA 16600 Practice Questions over 12.3 and 12.4

1. Recall the area from $\theta = \alpha$ to $\theta = \beta$ inside a polar graph is $\int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$

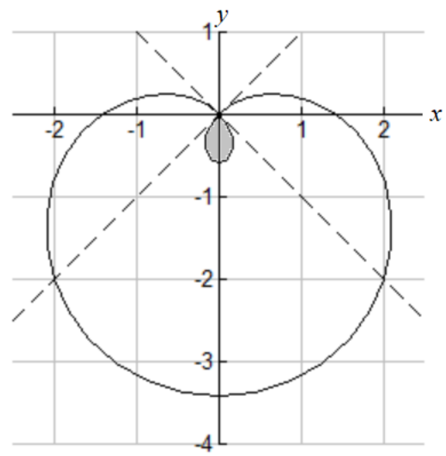
Find the exact area of the region inside one leaf of the 5-leaved rose $r = 5\cos 5\theta$.
You can use the FNINT command, but provide an exact area.



$$\int_{\square}^{\square} \boxed{} \, d\theta = \boxed{}$$

2. Set up the integral to calculate the area of the region inside the inner loop of the limaçon $r = \sqrt{2} - 2\sin\theta$. Use the FNINT command to find the area and approximate it the area to two decimal places.

To find the integration limits, find where $r = \sqrt{2} - 2\sin\theta = 0$ where $0 \le \theta < 2\pi$, since this will be where the inner loop starts and ends.
TIP: The dashed lines in the above graph are the polar equations $\theta = \alpha$ and $\theta = \beta$, where α and β are the lower and upper limits of integration. You can enter these values in your polar grapher as θmin and θmax to check that you have sketched only the inner loop.



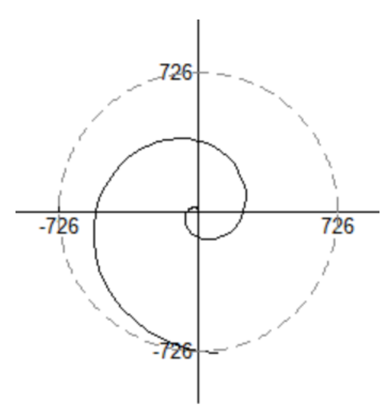
$$\int_{\square}^{\square} \boxed{} \, d\theta \approx \boxed{}$$

b. (+1 Bonus) What is the exact area, in terms of π? Show work for credit.

3. The arc length from $\theta = 0$ to $\theta = 11$ of a polar spiral $r = 6\theta^2$ is given by

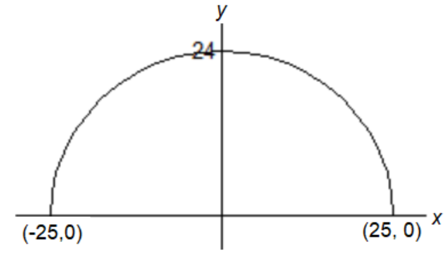
$\int_0^{11} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$. Calculate the arc length correct to the nearest whole number.

You can use the FNINT command. Round to the nearest whole number.



$$\int_0^{11} \sqrt{\boxed{}} \, d\theta \approx \boxed{}$$

4. The ceiling of a building is an ellipse with the dimensions shown (dimension feet). A person standing at c units from the y-axis that marks the center of the ellipse is able to hear the whispers of those standing the same distance on the other side of the y-axis. Assume both the person listening and those who are speaking are in the same vertical plane.

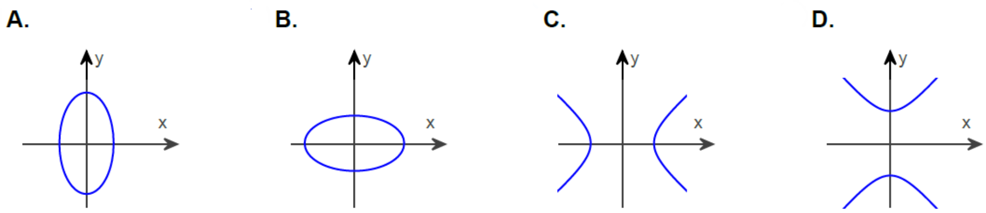


a. Write the formula of an ellipse with RUN = 25 and RISE = 24.
Your formula should be for a full ellipse, not the semi-ellipse shown.

b. What is c? Report a positive value.

5. Consider the conic section. $\dfrac{y^2}{25} - \dfrac{x^2}{36} = 1$.

a. Select which of these looks most like the graph?

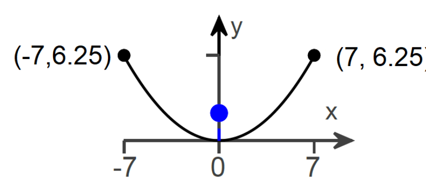


b. Report the vertices. ________________________________

Report the focal points as **exact** values: __________________

If the conic section is a hyperbola, report the asymptotes. Otherwise leave blank. __________________

6. A satellite dish is in the shape of a parabolic surface. Signals coming from a satellite strike the surface of the dish and are reflected to the focus, where the receiver is located. The satellite dish has a diameter of 14 feet and a depth of 6.25 feet.



a. Report the equation of the parabola. ____________________

b. How far from the base of the dish should the receiver be placed?

7. The vertices of a hyperbola centered at the origin are at the points (6,0) and (−6,0). Its asymptotes are $y = \pm\dfrac{1}{2}x$. Which of these is its equation?

A. $\dfrac{x^2}{4} - \dfrac{y^2}{2} = 1$ B. $\dfrac{x^2}{4} - \dfrac{y^2}{1} = 1$ C. $\dfrac{x^2}{2} - \dfrac{y^2}{1} = 1$ D. $\dfrac{x^2}{36} - \dfrac{y^2}{9} = 1$ E. $\dfrac{x^2}{6} - \dfrac{y^2}{3} = 1$

F. $\dfrac{x^2}{36} - \dfrac{y^2}{18} = 1$ G. None of these