Practice Questions from Appendix C Complex Numbers

1. a. Plot and **label** the complex numbers.

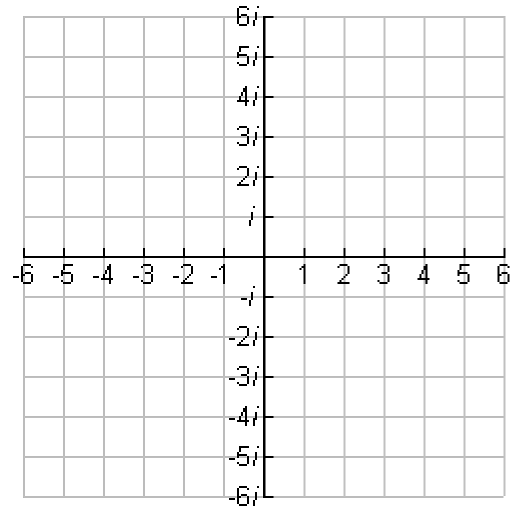
$z_1 = 5 - 5i$

$z_2 = -3i$

$z_3 = 2\sqrt{2}e^{i3\pi/4}$

$z_4 = 4e^{7\pi i}$

$re^{(i\theta)} = r(\cos\theta + i\sin\theta)$



b. Write z_1 and z_2 in exponential form $re^{i\theta}$, where r and θ are exact real numbers (and θ is in radians).
Hint: Part (a) may help.
(There are many correct answers for θ; however, report exact **radians** please.)

$z_1 = 5 - 5i$

r = ______________

θ = ______________ (Radians!)

Exponential form $re^{i\theta}$ of z_1 is ______________

$z_2 = -3i$

r = ______________

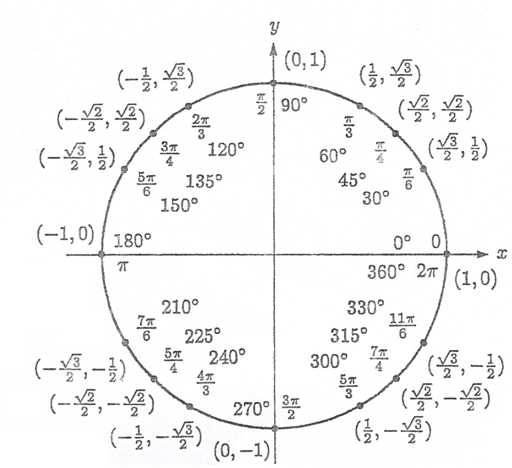
θ = ______________ (Radians!)

Exponential form $re^{i\theta}$ of z_2 is ______________

c. Write z_3 and z_4 in rectangular form $a + bi$, where a and b are real numbers.

$z_3 = 2\sqrt{2}e^{i3\pi/4} = \square + \square\, i$

$z_4 = 4e^{7\pi i} = \square + \square\, i$



2.a. Write $(2e^{i\pi/3})^4$ in the exponential form $re^{i\theta}$, where θ is exact and in radians.

r = ______

θ = ______ (Radians!)

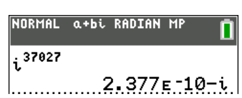
Exponential form $re^{i\theta}$ of $(2e^{i\pi/3})^4$ is______________

b. Write $(2e^{i\pi/3})^4$ in rectangular form $a + bi$, using exact values.

$\left(2e^{(i\frac{\pi}{3})}\right)^4 = \square + \square \cdot i$

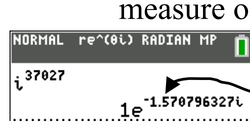
3. Consider the complex number i^{37027}.

a. A student uses a calculator to try to write the number in rectangular form $a + bi$, where a and b are real numbers. See the screen below.
What should the exact answer really be?
Report the exact answer in rectangular form $a + bi$:



$i^{37027} = \square + \square \cdot i$

b. When trying to write the number in polar form $re^{i\theta}$ where r and θ are real numbers, a student sees the screen below. What is the exact radian measure of the angle θ on the screen? (It involves π.)



θ = __________

c. Report the location of i^{37027} in the complex plane.
A. the positive real axis
B. the positive imaginary axis
C. the negative real axis
D. the negative imaginary axis

4. If the complex number z is represented by a vector, describe how to construct the vector u which is the complex number z multiplied by the number $re^{i\theta}$, i.e, $u = z \cdot re^{i\theta}$.

5. Report all the fourth roots of the number -1 and sketch them on the complex plane. Write them in the form $r\text{cis}\theta$ (using radians) and in rectangular form $a + bi$ (exact values please).

6. Report all the fourth roots of the number $-64i$ and sketch them on the complex plane. Write them in the form $r\text{cis}\theta$ (using degrees) and in rectangular form $a + bi$ (exact values please).

7. Report all the third roots of the number $8i$ and sketch them on the complex plane. Write them in the form $r\text{cis}\theta$ (using radians) and in rectangular form $a + bi$ (exact values please).

8. Consider the complex geometric series $f(z) = \sum_{k=0}^{\infty} 50z^k = 50 + 50z + 50z^2 + 50z^3 + \ldots$ which converges on $|z| < 1$. Report the value of $f\left(\frac{3i}{4}\right) = \sum_{k=0}^{\infty} 50\left(\frac{3i}{4}\right)^k$.

a. We separate even powers of $\frac{3i}{4}$ and odd powers of $\frac{3i}{4}$.

$$f\left(\frac{3i}{4}\right) = \sum_{k=0}^{\infty} 50\left(\frac{3i}{4}\right)^k = 50\left(1 + \left(\frac{3i}{4}\right)^2 + \left(\frac{3i}{4}\right)^4 + \left(\frac{3i}{4}\right)^6 + \ldots\right) + 50\left(\left(\frac{3i}{4}\right)^1 + \left(\frac{3i}{4}\right)^3 + \left(\frac{3i}{4}\right)^5 + \left(\frac{3i}{4}\right)^7 + \ldots\right)$$

First simplify powers of ***i***.
Then combine real parts in the first row and imaginary parts in the second row.
Then factor out 50 in the first row and $50 \cdot \frac{3i}{4}$ in the second row. Enter **real** numbers in each box.
You can write the real numbers as powers of $\frac{3}{4}$.

$$f\left(\frac{3i}{4}\right) = 50\left(1 + \boxed{} + \boxed{} + \boxed{} + \ldots\right) + 50 \cdot \frac{3i}{4}\left(1 + \boxed{} + \boxed{} + \boxed{} + \ldots\right)$$

b. The geometric series $1 + \left(\frac{3i}{4}\right)^2 + \left(\frac{3i}{4}\right)^4 + \left(\frac{3i}{4}\right)^6 + \ldots$ has $a = 1$ and $r = \boxed{}$ and sum equal to $\boxed{}$..

The geometric series here has $a = 1$ and $r = \boxed{}$ and sum equal to $\boxed{}$.

c. Combining, we have $f\left(\frac{3i}{4}\right) = \sum_{k=0}^{\infty} 50\left(\frac{3i}{4}\right)^k = \boxed{} + \boxed{}\, i$ (Insert integers in the boxes.)